Mathematical challenges for able pupils

in Key Stages 1 and 2

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Introduction

This book supplements the *National Literacy and Numeracy Strategies: guidance on teaching able children*, published in January 2000. Its purpose is to help primary teachers cater for pupils who are more able in mathematics and likely to exceed the expected standards for their year group.

Mathematically able pupils are in every school and among all ethnic and socio-economic groups.

They typically:

- grasp new material quickly;
- are prepared to approach problems from different directions and persist in finding solutions;
- generalise patterns and relationships;
- use mathematical symbols confidently;
- develop concise logical arguments.

The *Framework for teaching mathematics from Reception to Year 6* covers the National Curriculum for Key Stages 1 and 2 from pre-level 1 up to level 4 and parts of level 5. The draft *Framework for teaching mathematics: Year 7*, published in March 2000, is based mainly on work at level 5. The yearly teaching programmes in the *Framework* are expressed as 'targets for the majority of pupils in the year group'. Many able pupils will progress more quickly through these programmes and will need extension and enrichment activities in mathematics.

This book addresses class organisation, planning and teaching through answers to commonly asked questions.

The puzzles and problems in the second part of this book can be photocopied for use in schools in England participating in the National Numeracy Strategy. The puzzles and problems are also available on the National Numeracy Strategy website (see page 9).

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Some questions answered

How should we organise within the school?

Within the class

You will probably teach able pupils in their own class for their daily mathematics lesson. They will cover the same topics as their peers but at a level to match their abilities. You can stretch them through differentiated group work, harder problems for homework and extra challenges – including investigations using ICT – which they can do towards the end of a unit of work when other pupils are doing consolidation exercises. The planning and structure of the National Numeracy Strategy address the needs of all pupils and help you to manage classes with wide-ranging attainment groups. Each pupil, very able or less able, needs to be part of one of these groups for at least some of the time and not restricted totally to individual working.

With an older year group

Pupils who are exceptionally gifted in many subjects, and who are sufficiently mature, may be promoted to work with an older age group. For example, you could timetable Year 3 and Year 4 mathematics lessons at the same time. An exceptionally gifted pupil in Year 3 could be taught the subject with the Year 4 class and benefit from discussion with other pupils working at a similar level.

Setting

Larger schools with parallel classes sometimes deal with a range of attainment by organising 'ability sets' for mathematics lessons. The advantage is that your planning can be easier if the attainment gap in a class is not too wide. You could set across, say, Years 5 and 6, if both years are timetabled for their mathematics lessons at the same time, although you need to ensure that when Year 5 pupils move into Year 6 they do not simply repeat the previous year's activities. Any setting arrangements need to be flexible to allow easy transfer of pupils between sets. The success of setting depends on very careful monitoring, close teamwork and co-operative planning among teachers to make sure that expectations for all pupils are suitably high and that lower expectations are not justified simply because pupils are in a 'lower set'.

How can I adapt my termly planning?

In Key Stage 1, the aim is to provide a firm foundation in mathematics for all pupils. The needs of able pupils are best served through an accelerated programme, spending the same amount of time as other pupils, but going further with each topic. This approach should be supplemented by a more investigative approach to learning. The table overleaf shows how part of an autumn term plan for an 'average' Year 2 class has been modified to cater for able pupils, by including objectives from the teaching programmes for Years 3 and 4. Each unit of work concentrates on the same topic to help you to manage the necessary differentiation. Enrichment activities encourage pupils to develop their skills in problem solving and reasoning. The suggestions in the plan overleaf have been drawn from the puzzles and problems in the second part of this book.

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Medium-term plan: Year 2
Part of autumn term with extension and enrichment work

Unit (days)	Topic Objectives: children will be taught to…	Extension Objectives from Y3/4	Enrichment suggestions
1 (3)	**Counting, properties of numbers and number sequences**		
	Say the number names in order to at least 100.		Snakes and ladders (no. 4)
	Count reliably up to 100 objects by grouping them, for example in tens.	Estimate up to 100 objects.	Cross-road (no. 17)
	Count on or back in ones or tens from any two-digit number.	Extend to three-digit numbers.	
	Recognise two-digit multiples of 10.	Recognise three-digit multiples of 10.	
	Count in hundreds from and back to zero.		
2–4 (15)	**Place value, ordering, estimating, rounding**		
	Read and write whole numbers from 0 to 100 in figures and words.	Read/write numbers to 1000/10 000.	
	Know what each digit in a two-digit number represents, including 0 as a place-holder.	Extend to three-digit numbers.	Ben's numbers (no. 24)
	Partition two-digit numbers into a multiple of ten and ones (TU). Use the = sign.	Partition three-digit numbers.	
	Say the number that is 1 or 10 more or less than any given two-digit number.		
	Understanding addition and subtraction		
	Understand the operations of addition and subtraction; recognise that addition can be done in any order, but not subtraction.		Number lines (no. 11)
	Use the +, – and = signs to record mental calculations in a number sentence.		
	Mental calculation strategies (+/–)		
	Put the larger number first.		
	Count on or back in tens or ones.	Count on/back in 100s.	
	Identify near doubles, using doubles already known.		
	Add/subtract 9 or 11 by adding/subtracting 10 and adjusting by 1.	Extend to 19, 29…; 21, 31…; 18, 22…	
	Money and 'real life' problems		
	Recognise all coins. Find totals.	Give change.	
	Choose an appropriate operation and calculation strategy to solve simple word problems.	Solve word problems.	Ride at the fair (no. 8)
	Explain methods orally…	…and in writing.	Gold bars (no. 7)
	Making decisions, checking results		
	Check sums by adding in a different order.		

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In Key Stage 2, the accelerated programme can continue, as well as including more challenging problems and extended pieces of work. Termly plans should still ensure that able pupils are taught a broad, balanced mathematics curriculum.

The table below illustrates part of a 'typical' Year 6 termly plan for mathematics with enhanced provision for able pupils. The extra objectives are drawn from the Year 7 draft *Framework*. The programme has been supplemented with enrichment activities that develop higher order thinking and problem solving skills. These 'challenges' are best linked to the main class topic.

The work on page 14 of this book illustrates how the main Year 6 teaching programme on multiplication can be supplemented for able pupils. While most pupils are consolidating their skills in using a written method for multiplication, able pupils might investigate other written methods for long multiplication.

Medium-term plan: Year 6
Part of autumn term with extension and enrichment work

Unit (days)	Topic Objectives: children will be taught to…	Extension Objectives from Y7	Enrichment suggestions
1 (3)	**Place value, ordering and rounding** Multiply and divide decimals by 10 or 100, and integers by 1000, and explain the effect.	Understand and use decimal notation and place value.	
	Using a calculator Develop calculator skills and use a calculator effectively.		Millennium (no. 81)
2–3 (10)	**Understanding multiplication and division** Understand and use the relationships between the four operations, and the principles of the arithmetic laws.	Express simple functions at first in words and then in symbols, and use simple function machines.	Maze (no. 62)
	Mental calculation strategies (× and ÷) Use related facts and doubling or halving: e.g. halve an (even) number, double the other; multiply by 25, e.g. by ×100, then ÷4. Extend mental methods (to decimals).		Shape puzzle (no. 72) Make five numbers (no. 61)
	Pencil and paper procedures (× and ÷) Approximate first. Use informal pencil and paper methods to support, record or explain × and ÷.		Alternative multiplication (see p. 14)
	Extend written methods to ThHTU × U and short multiplication involving decimals.	Extend to decimals with 2 d.p.	
	Money and 'real life' problems Use all four operations to solve money or 'real life' problems.		Spendthrift (no. 79)
	Choose appropriate operations/calculation methods. Explain working.		Franco's fast food (no. 67)
	Making decisions, checking results Check by estimating. Use inverse operation, including with a calculator.		Flash Harry (no. 64)

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How can I use the 'extra' week each term?

For all year groups the optional termly planning grids leave a week unallocated each term. This 'extra' week can be used in different ways. Some pupils may need to consolidate and develop a previous piece of work. Able pupils could, after an introduction by you, do a sustained piece of extension work. This might involve some research and investigation, and could be linked to the main teaching programme for the class or could be a new topic. It could draw on subjects other than mathematics. As an example, the work on palindromic numbers on pages 12–13 of this book might be suitable for Year 4 pupils.

How can I use the three-part lesson?

In the **oral/mental part** of the lesson, you can direct some questions towards the most able pupils, just as you can direct some specifically towards the children who find mathematics difficult. Able pupils can also contribute by suggesting and explaining alternative methods of calculation.

In the **main part** of the lesson you will often introduce a new topic with some direct teaching of the whole class. You will consolidate previous ideas and develop and use the correct mathematical language. For able pupils, the amount of practice and consolidation needed is less than that required by other pupils. Within whole-class teaching, you can set different tasks for pupils to undertake, for example:

- a common task, starting from the common experience of pupils, leading to different outcomes – this is typical of open investigations;
- a stepped task that helps pupils build on their own learning strategies – each step needs to be relevant and purposeful, and able pupils can omit earlier steps;
- separate tasks for each group of pupils, but linked to a common theme.

You should give all pupils opportunities to apply their mathematical knowledge. Able pupils can often move quickly beyond basic knowledge and skills and begin to use these in a range of contexts. Problems need not involve difficult mathematics but may require insight, reasoning and higher order thinking skills in order to reach a solution.

The **plenary session** gives you opportunities to extend as well as consolidate work. Methods of solution can be compared and explanations shared.

Homework can provide the opportunity for pupils to tackle challenging questions and puzzles. The results can form the basis of the next lesson with either the whole class or a group. Opportunities also exist for pupils to read about mathematical topics.

In all parts of the lesson, the quality of questioning is crucial in helping pupils develop mathematical ideas and improve their thinking skills. The National Numeracy Strategy *Mathematical vocabulary* contains guidance on types of questioning appropriate to all parts of the mathematics lesson. The range of questioning should include recalling and applying facts, hypothesising and predicting, designing and comparing procedures, interpreting results and applying reasoning. You can use some open questions to allow more pupils to respond at their own level. Such questions often provide a greater challenge for able pupils, who can be asked to think of alternative solutions and, in suitable cases, to list all the different possibilities: 'Can you suggest another method you might have used?' 'Would it work with different numbers?' 'How do you know you have included all the possibilities?'

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Where can I find enrichment activities to develop pupils' thinking skills?

Puzzles and problems in this book

The second part of this book contains puzzles and problems. These are accessible to a wide range of pupils. There are three sections covering Years 1 and 2, Years 3 and 4, and Years 5 and 6. The problems are intended to challenge pupils and extend their thinking. While some of them may be solved fairly quickly, others will need perseverance and may extend beyond a single lesson. Pupils may need to draw on a range of skills to solve the problems. These include: working systematically, sorting and classifying information, reasoning, predicting and testing hypotheses, and evaluating the solutions.

Many of the problems can be extended by asking questions such as: 'What if you tried three-digit numbers?' 'What if there were more boxes?' 'What if you used triangles instead of squares?' Problems can also be extended by asking pupils to design similar problems of their own to give to their friends or families.

Learning objectives appropriate to each problem are indicated so that you can target problems by integrating them into your main teaching programme.

Solutions are given at the end of the book.

Extended tasks, problems or investigations within/beyond the main curriculum

Resources that schools may find useful include:

- books of investigations and 'open' problems;
- mathematics magazines and booklets produced for pupils;
- mathematical posters and topic books that stimulate discussion and investigation;
- computer access to the Internet;
- calculators to solve challenging and investigative activities;
- software;
- 'general' books on mathematics, e.g. history of mathematics, biographies of mathematicians.

Competitions

The Mathematical Association introduced a Primary Mathematics Challenge in November 1999. This competition will run annually from 2000.

Websites

The puzzles and problems in this book are available on the National Numeracy Strategy website:

www.standards.dfee.gov.uk/numeracy/

The Maths Year 2000 website contains puzzles and problems, and links to a number of other mathematical websites:

www.mathsyear2000.org

The nrich website also provides a regular supply of problems:

www.nrich.maths.org.uk

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Which National Numeracy Strategy materials support the teaching of able pupils?

Framework for teaching mathematics from Reception to Year 6

The organisation of teaching objectives and the supplement of examples signal the progression in topics, clarifying the links between the teaching programmes of each year group. Teachers who are planning work for able pupils should give particular attention to the introductory section on laying the foundations for algebra.

Framework for teaching mathematics: Year 7 (draft issued in March 2000)

This extends the original *Framework* and is based mainly on work at level 5.

Mathematical vocabulary

The introduction to this booklet contains useful references to questioning techniques with examples of the types of question that help to extend children's thinking.

Sample termly plans (on CD issued with December 1999 Professional development materials 3 and 4)

These may help you to identify 'What comes next?' in a particular topic. By looking ahead one term, two terms or even further, you can incorporate objectives into your present plan as extension work.

Sessions from the five-day training course for intensive schools

- **Problem solving with challenges and simplifications:** This illustrates how activities linked to problem solving and reasoning can challenge able pupils.
- **Using a calculator:** Although the activities are designed for teachers, some of them can be used to extend able pupils.
- **Laying the foundations for algebra:** This contains a range of examples involving reasoning and explanation.
- **Fractions, decimals, percentages, ratio and proportion:** Able pupils can develop this work, especially the inter-relationships and examples involving ratio and proportion.
- **Shape and space:** This gives some background work on transformations. Reflections, translations and rotations are covered separately as well as an introduction to combining transformations.
- **Graphs and charts:** Able pupils can be encouraged to tackle problems that require data collection and analysis. Particular emphasis needs to be given to the interpretation of results.

Professional development materials 3 and 4 (issued to all schools in December 1999)

This pack covers many of the same topics as the five-day course materials. Chapters that may be of particular relevance are:

- Solving word problems
- Fractions, decimals, percentages, ratio and proportion
- Shape and space
- Calculators
- Graphs and charts

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Where else can I get help?

Other sources of support include:

- Local education authority advisory services
- Local universities
- The Mathematical Association
  259 London Road
  Leicester
  LE2 3BE
- The Association of Teachers of Mathematics
  7 Shaftesbury Street
  Derby
  DE23 8YB
- The National Association of Able Children in Education
  NAACE National Office
  Westminster College
  Harcourt Hill
  Oxford
  OX2 9AT
- The National Association of Gifted Children
  NAGC
  Elder House
  Milton Keynes
  MK9 1LR
- The Royal Institution
  21 Albemarle Street
  London
  W1X 4BS

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Activity examples

Palindromic numbers (Year 4)

Meanings

Look up the meaning of ‘palindrome’ in a dictionary.

Words can be palindromic, for example ‘madam’.
Dates can be palindromic too, for example 17.8.71.
Can you think of some more examples?

Palindromic numbers

8, 33, 161, 222 and 2998992 are examples of palindromic numbers.

- How many palindromic numbers are there between:
  0 and 100? 100 and 200? 200 and 300? 300 and 400?
  0 and 1000? 1000 and 1100? 1100 and 1200? 1300 and 1400?
- Can you work out how many palindromic numbers there are between 0 and 2000?
  What about between 0 and 10000?

Backwards and forwards

Start with a two-digit number, for example:	32
Reverse it and add the result to the original number:	32 + 23 = 55
The result is palindromic after one reversal.	55
Now try it with another two-digit number, such as:	57
Reverse it and add the result to the original number:	57 + 75 = 132
Reverse and add again:	132 + 231 = 363
This time the result is palindromic after two reversals.	363

- Can you find two-digit numbers that are palindromic after one reversal?
  After two reversals? After three reversals? After more than three reversals?

  The numbers 89 and 98 take 24 reversals!
- Investigate the same process with three-digit numbers.

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Continue the pattern

Continue each of these patterns.
In each case, describe what you notice.

- $1 \times 9 + 2 =$
  $12 \times 9 + 3 =$
  $123 \times 9 + 4 =$
  and so on.
- $11 \times 11 =$
  $111 \times 111 =$
  $1111 \times 1111 =$
  and so on.
- $11 \times 11 =$
  $11 \times 11 \times 11 =$
  $11 \times 11 \times 11 \times 11 =$
  and so on.

Questions with palindromic answers

Try to make up some questions with palindromic answers.
You might need to work out what the answers should be first!

Hints and solutions (for teachers)

One-digit palindromes: 1, 2, 3, …, 9 are palindromic, so there are 9 palindromic one-digit numbers. (But some people might want to include 0 as well!)

Two-digit palindromes: 11, 22 and so on are palindromic, so there are 9 numbers.

Three-digit palindromes: 1◆1 where ◆ stands for the digits 0 to 9
2◆2
and so on.

There are 90 three-digit palindromes.

Four-digit palindromes: between 1000 and 1100 there is only 1001,
between 1100 and 1200 there is only 1111,
and so on.

Between 1000 and 2000 there are 10 palindromic numbers.

Here are some other calculations that have palindromic answers:

22×11 33×11 44×11 407×3 1408×3 143×7

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Alternative multiplication (Year 6)

Look at these methods for long multiplication.
Can you work out what is happening? Why do they work?

Try them for yourself using other numbers.

Which method do you like best?

Multiplication method 1

27×43

$1 \times 43 =$	43	43
$2 \times 43 =$	86	86
$4 \times 43 =$	172	
$8 \times 43 =$	344	344
$16 \times 43 =$	688	688

So $27 \times 43 =$ 1161

14×78

$1 \times 78 =$	78	
$2 \times 78 =$	156	156
$4 \times 78 =$	312	312
$8 \times 78 =$	624	624

So $14 \times 78 =$ 1092

Multiplication method 2

27×43

$27 \times$	43	43
$13 \times$	86	86
$6 \times$	172	
$3 \times$	344	344
$1 \times$	688	688

So $27 \times 43 =$ 1161

38×47

$38 \times$	47	
$19 \times$	94	94
$9 \times$	188	188
$4 \times$	376	
$2 \times$	752	
$1 \times$	1504	1504

So $38 \times 47 =$ 1786

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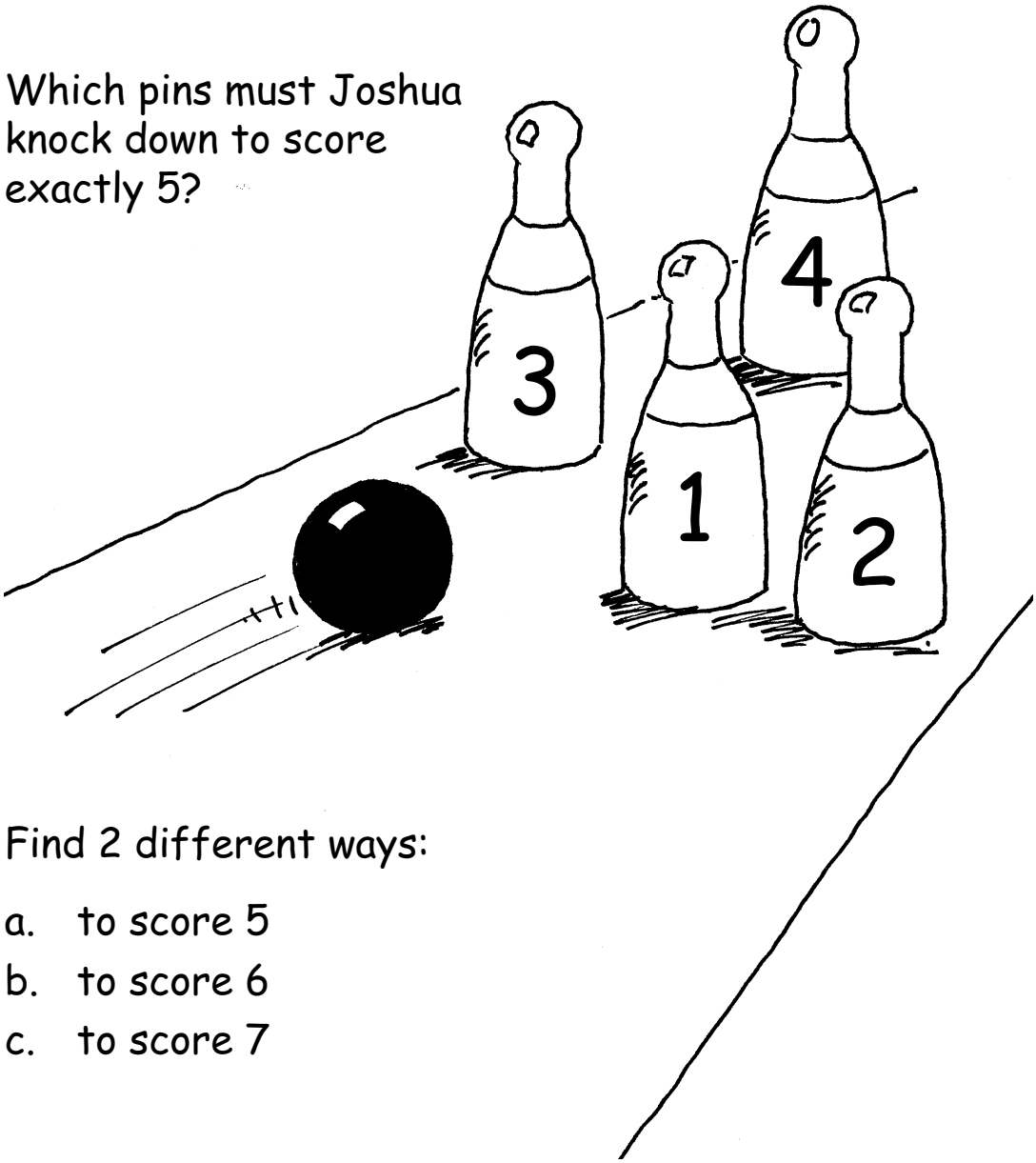
Puzzles and problems for Years 1 and 2

Four-pin bowling

Which pins must Joshua knock down to score exactly 5?



Find 2 different ways:

a. to score 5

b. to score 6

c. to score 7

1

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 10.

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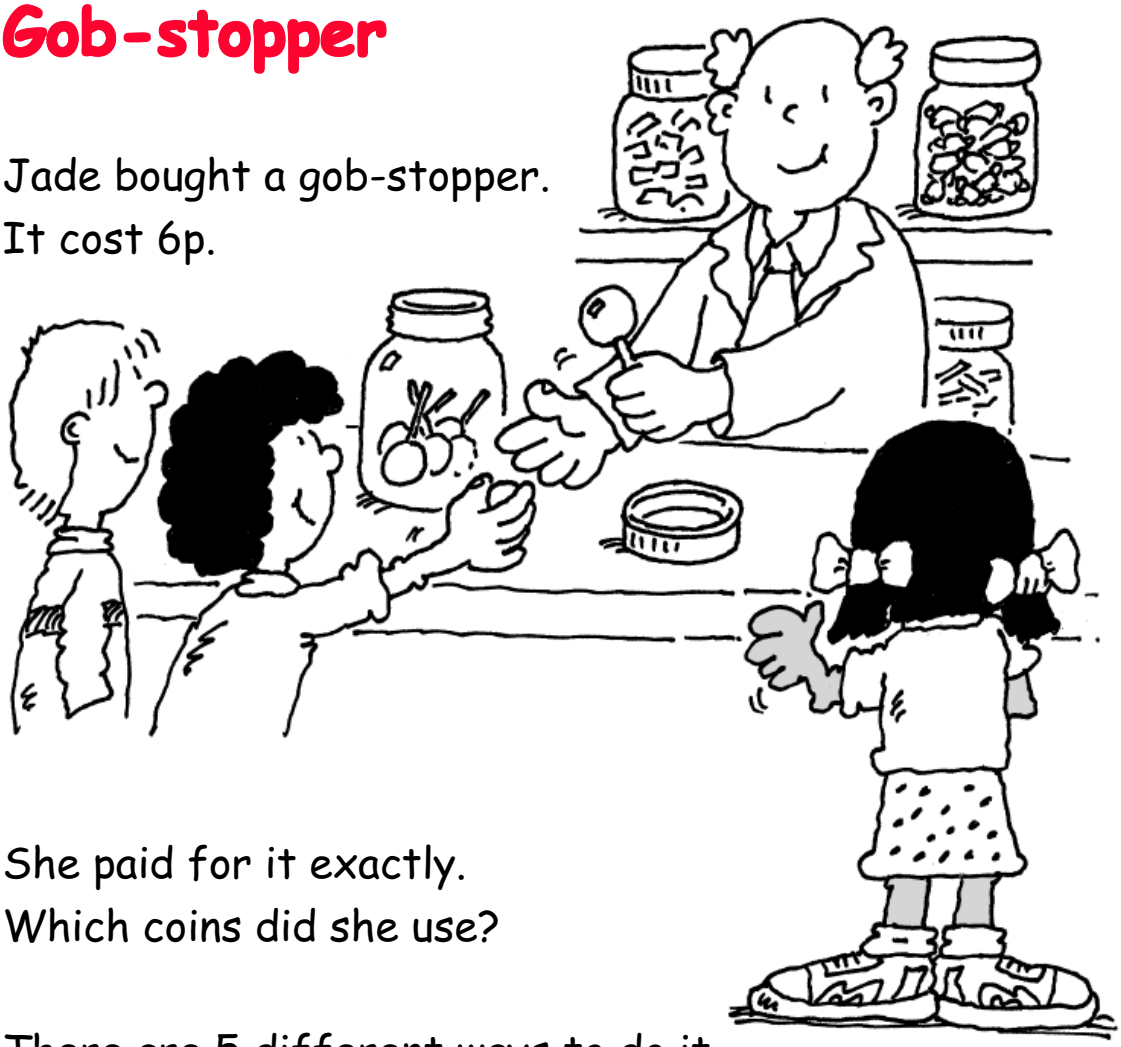
Gob-stopper

Jade bought a gob-stopper.
It cost 6p.



She paid for it exactly.
Which coins did she use?

There are 5 different ways to do it.
Find as many as you can.

What if the gob-stopper cost 7p?

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 10.
Find totals, give change, and work out which coins to pay.

2

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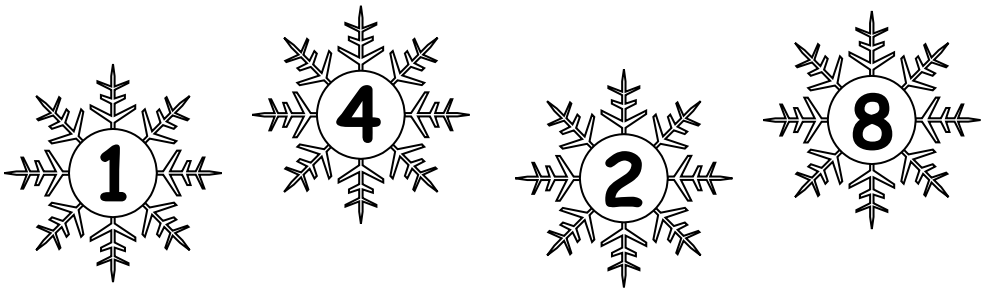
Pick a pair

Choose from these numbers.



1. Pick a pair of numbers.
   Add them together.
   Write the numbers and the answer.

   Pick a different pair of numbers.
   Write the numbers and the answer.

   Keep doing it.
   How many different answers can you get?

2. Now take one number from the other.
   How many different answers can you get now?

3

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 10.

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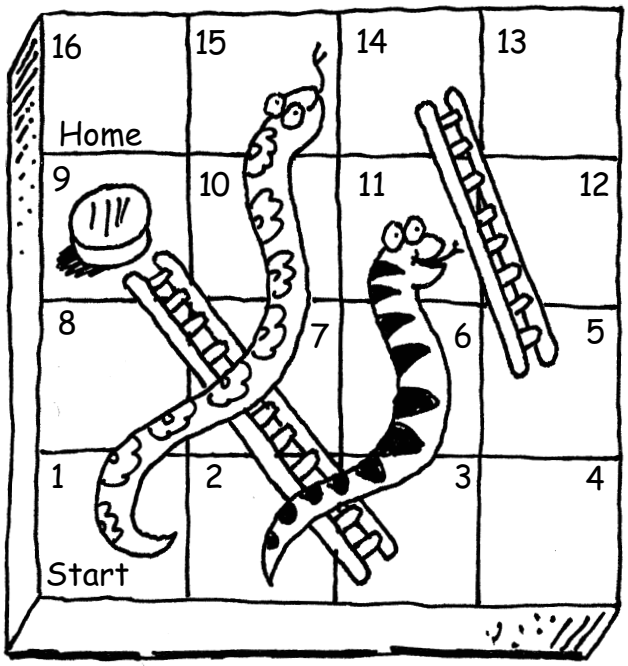
Snakes and ladders



Your counter is on 9.

You roll a 1 to 6 dice.
After two moves you land on 16.

Find all the different ways you can do it.

Now think of other questions you could ask.

Teaching objectives

Solve mathematical problems or puzzles.
Count on from any small number.

4

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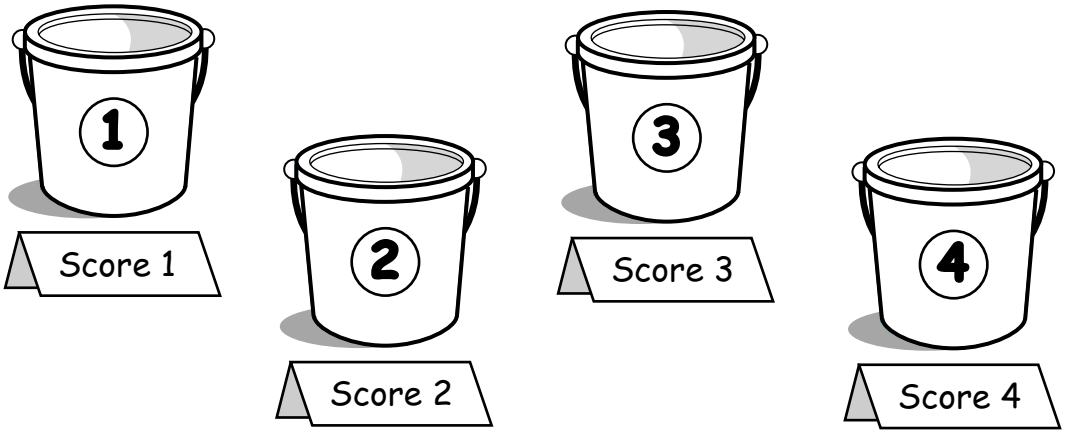
Bean-bag buckets

Dan threw 3 bean-bags.
Each bag went in a bucket.
More than one bag can go in a bucket.



1. What is the highest score Dan can get?
2. Find three ways to score 6.
3. Find three ways to score 9.
4. What other scores can Dan get?

5

Teaching objectives

Solve mathematical problems or puzzles.
Know addition facts up to 10.

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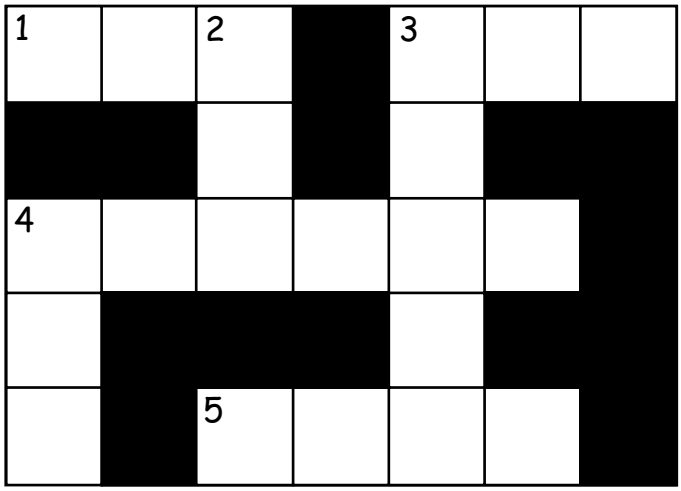
Crossword

Write the answers to this puzzle in words:
ONE, TWO, THREE, ...



Across

1. 7 – 5

3. 2 + 5 – 1

4. 4 + 4 + 4

5. 13 – 4

Down

2. 3 + 4 – 6

3. 9 – 2

4. 11 – 4 + 3

Teaching objectives

Solve mathematical problems or puzzles.
Use known number facts and place value to add and subtract mentally.
Read and write whole numbers.

6

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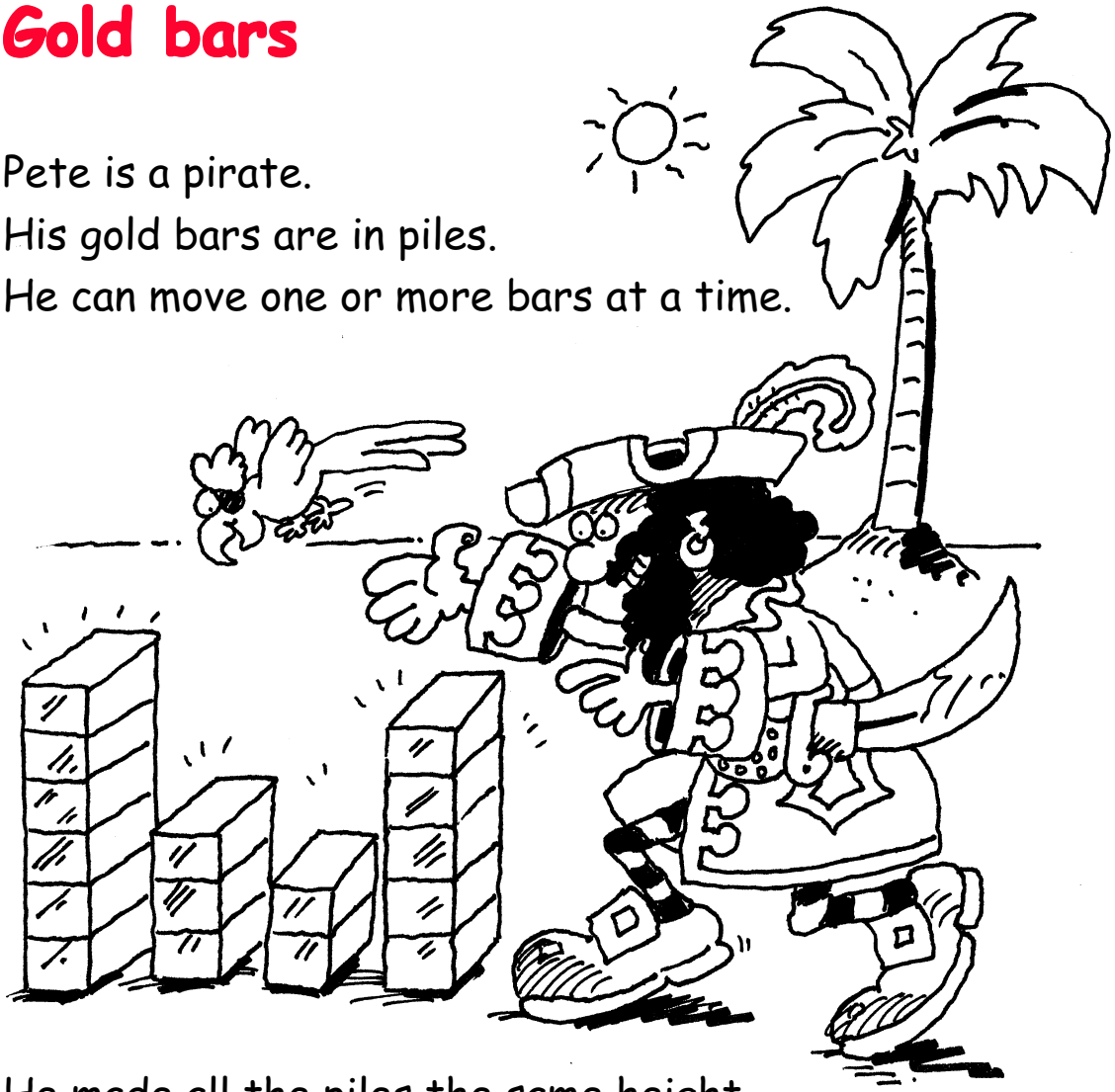
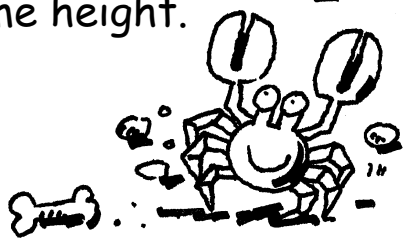
Gold bars

Pete is a pirate.
His gold bars are in piles.
He can move one or more bars at a time.



He made all the piles the same height.
He made just two moves.
How did he do it?



7

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

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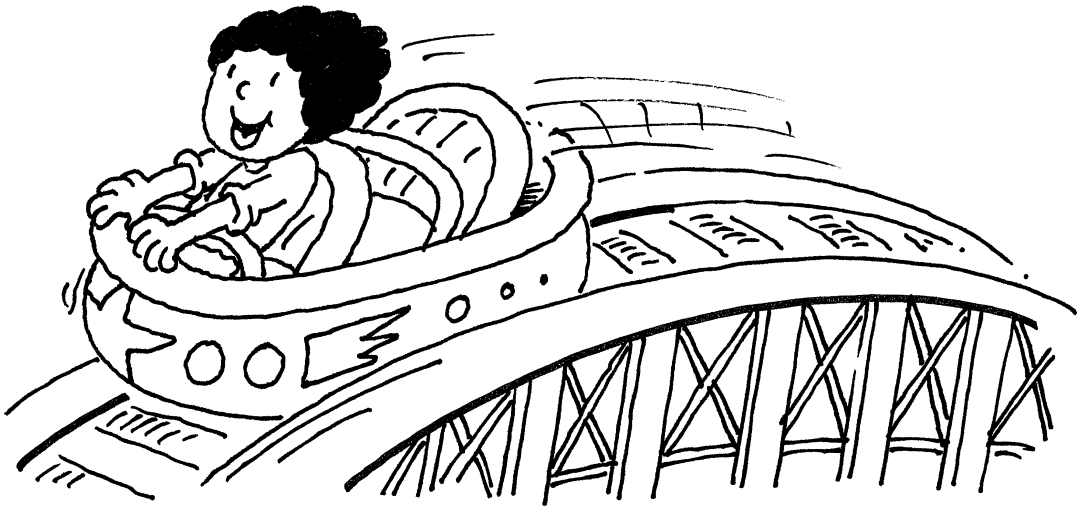
Ride at the fair



Lucy had a ride at the fair.
Her Mum asked Lucy to pay less than 20p towards it.

Lucy paid exactly three coins towards the ride.
How much did Lucy pay her Mum?

Find different ways to do it.

Teaching objectives

Solve mathematical problems or puzzles.
Find totals, give change, and work out which coins to pay.

8

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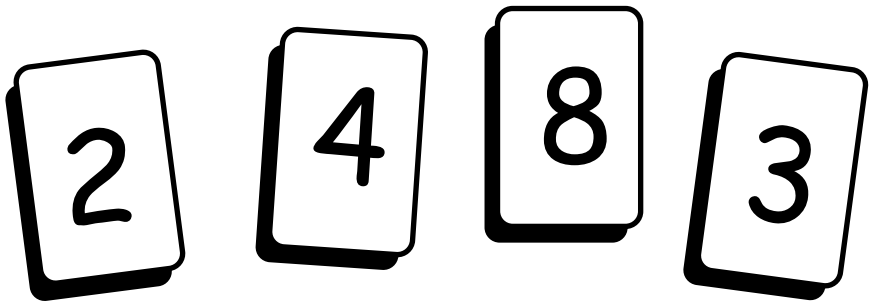
Sum up

Choose from these four cards.



Make these totals:

9
10
11
12
13
14
15

What other totals can you make from the cards?

9

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts to at least 10.
Add three small numbers mentally.

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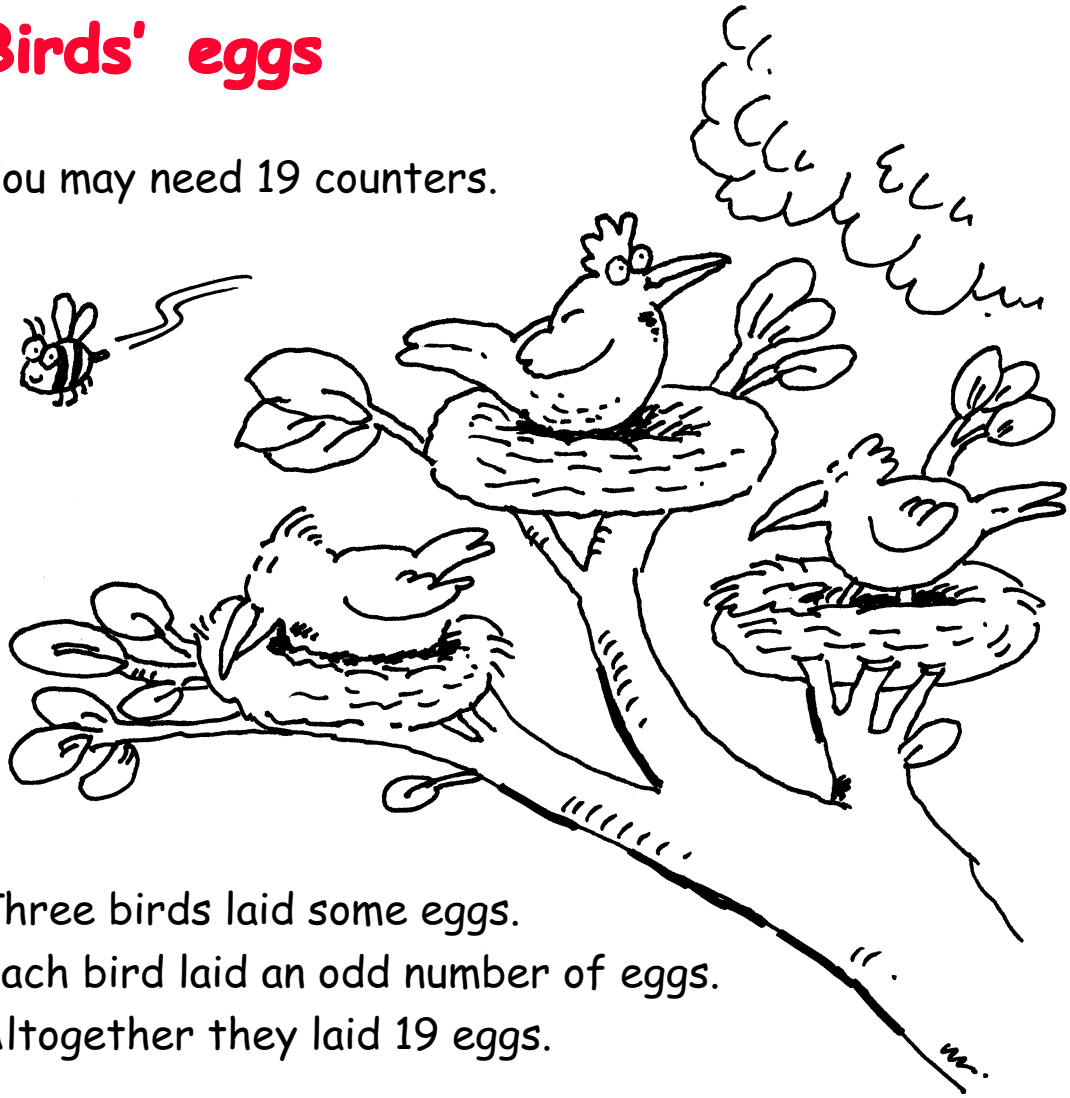
Birds' eggs

You may need 19 counters.



Three birds laid some eggs.
Each bird laid an odd number of eggs.
Altogether they laid 19 eggs.

How many eggs did each bird lay?
Find different ways to do it.

Teaching objectives

Solve mathematical problems or puzzles.
Recognise odd and even numbers.
Add three small numbers mentally.

10

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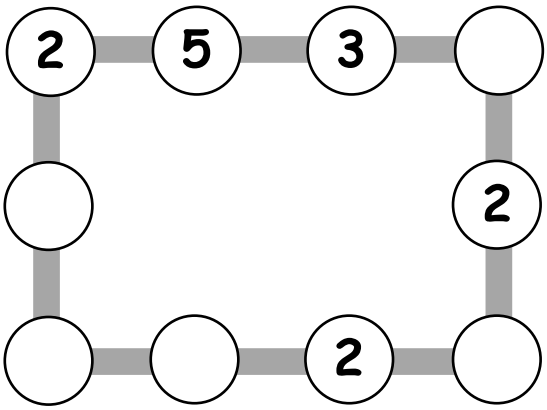
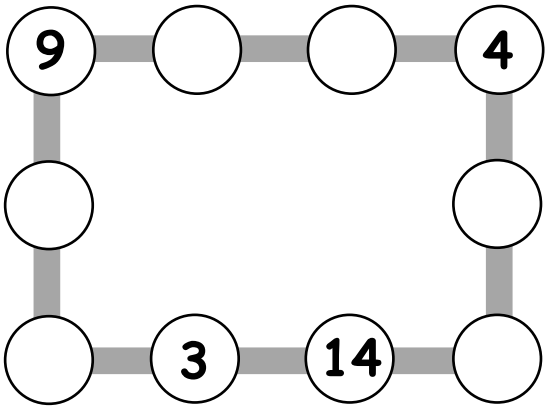
Number lines

1. Make each line add up to 16.



2. Make each line add up to 20.



3. Make up your own puzzle like this.
   Ask a friend to do it.

11

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 20.
Add three small numbers mentally.

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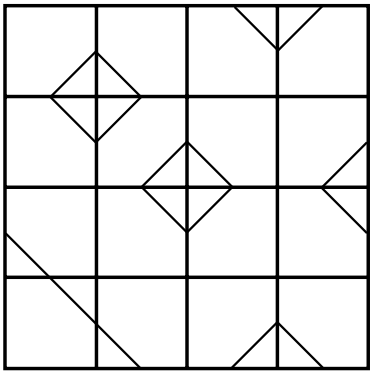
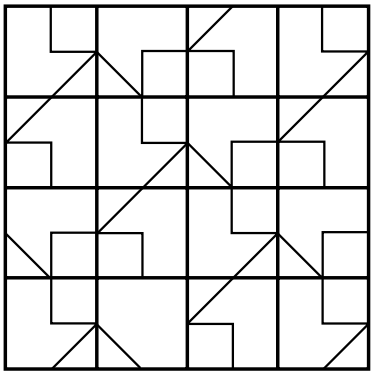
Odd one out

1. Here is a grid of 16 squares.
   One square is different from all the others.
   Mark it on the grid.



2. Now do this one.



Teaching objectives

Solve mathematical problems or puzzles.
Make and describe patterns and pictures.

12

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Line of symmetry

You need:
some squared paper,
a red pen, a green pen and a blue pen.

Gopal had six squares: two red, two green, two blue.
He put them in a line.
The squares made a symmetrical pattern.

red	blue	green	green	blue	red

Arrange six squares in a line.
Make two squares red, two green and two blue.
Make the line of squares symmetrical.

How many different lines can you make like this?

13

Teaching objectives

Solve mathematical problems or puzzles.
Begin to recognise line symmetry.
Solve a problem by sorting, classifying and organising information.

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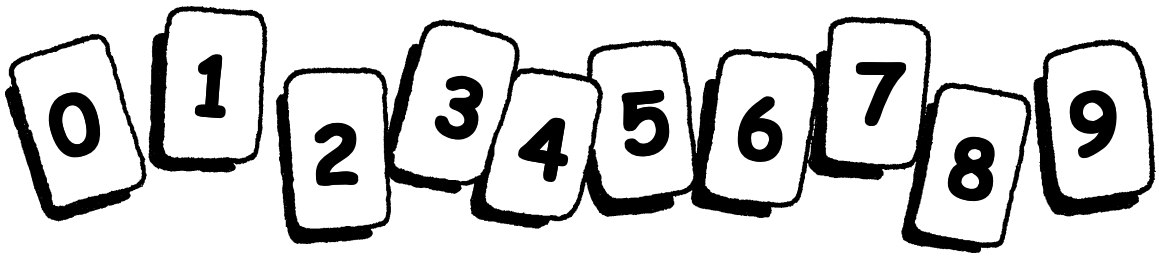
Card sharp

Take ten cards numbered 0 to 9.



1. Pick three cards with a total of 12.
   You can do it in 10 different ways.
   See if you can record them all.

2. Now pick four cards with a total of 12.
   How many different ways can you do it?

3. Can you pick five cards with a total of 12?

Teaching objectives

Solve mathematical problems or puzzles.
Know addition facts to at least 10.
Solve a problem by sorting, classifying and organising information.

14

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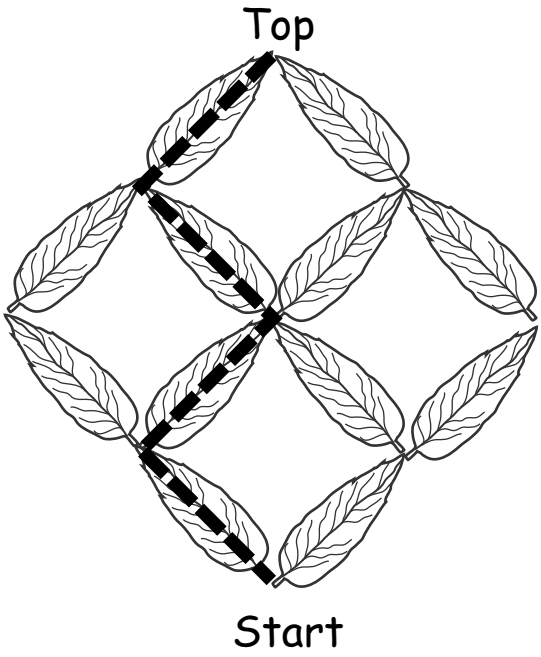
Jack and the beanstalk

Jack climbed the beanstalk.
He always went upwards.



He first did it like this: left, right, left, right.

Find five other ways that Jack can climb the beanstalk.

15

Teaching objectives

Solve mathematical problems or puzzles.
Recognise turns to the left or to the right.
Give instructions for moving along a route.

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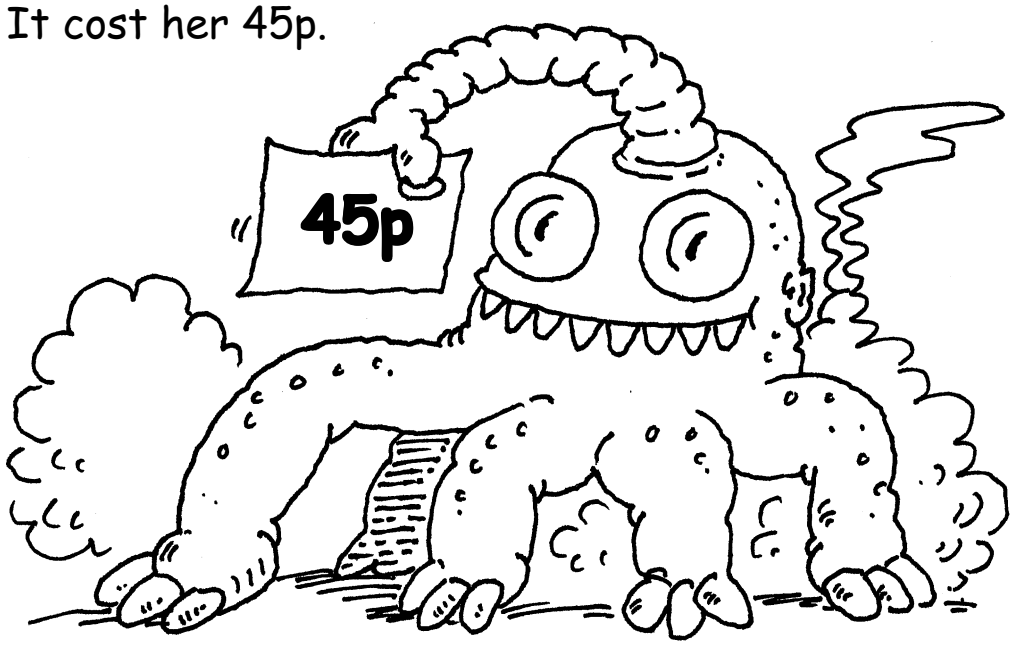
Monster

Alesha bought a monster using only silver coins.
It cost her 45p.



There are nine different ways to pay 45p exactly using only silver coins.
Find as many as you can.

What if the monster cost 50p?
How many different ways are there to pay now?

Teaching objectives

Solve mathematical problems or puzzles.
Find totals.
Work out which coins to pay.

16

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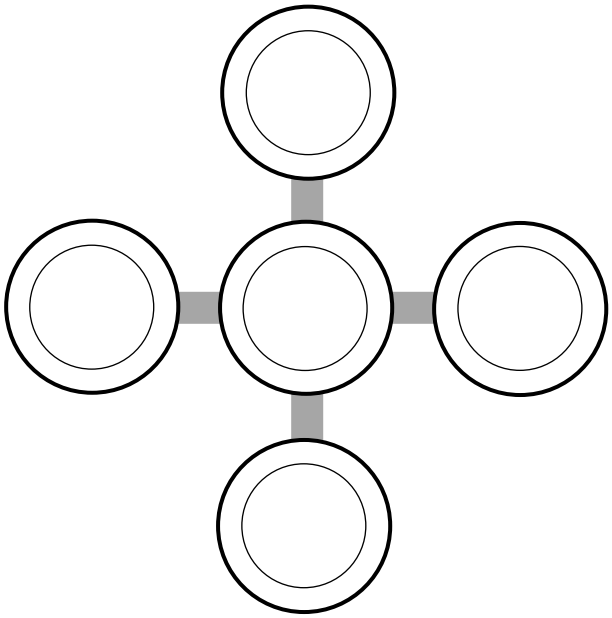
Cross-road

You need 5 paper plates and 15 counters.
Put the plates in a cross.



Use all 15 counters.
Put a different number on each plate.
Make each line add up to 10.

Do it again.
This time make each line add up to 8.

17

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 10.
Add three small numbers mentally.

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Fireworks

Emma had some fireworks.
Some made 3 stars.
Some made 4 stars.

Altogether Emma's fireworks made 19 stars.
How many of them made 3 stars?
Find two different answers.

What if Emma's fireworks made 25 stars?
Find two different answers.

Teaching objectives

Solve mathematical problems or puzzles.
Count on in steps of 3 or 4 from zero, or from any small number.

18

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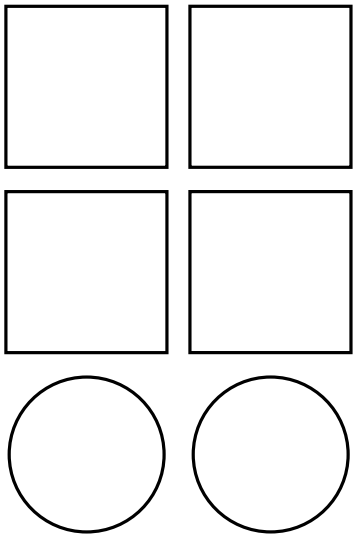
Coloured shapes

What colour is each shape?
Write it on the shape.



Clues

- Red is not next to grey.
- Blue is between white and grey.
- Green is not a square.
- Blue is on the right of pink.

19

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

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Ones and twos

Holly has six numbers, three 1s and three 2s.
She also has lots of + signs, x signs and = signs.

1 2 1 2 1 2

She is trying to make the biggest number possible.
Here are some she tried.

First try	Second try
1 x 2 = 2	1 + 2 + 1 + 2 + 1 + 2 = 9
1 x 2 = 2	
1 x 2 = 2	
2 + 2 + 2 = 6	

Can you beat Holly's score?

What if Holly had three 2s and three 3s?

Teaching objectives

Solve mathematical problems or puzzles.
Use known number facts to add mentally.
Carry out simple multiplication.

20

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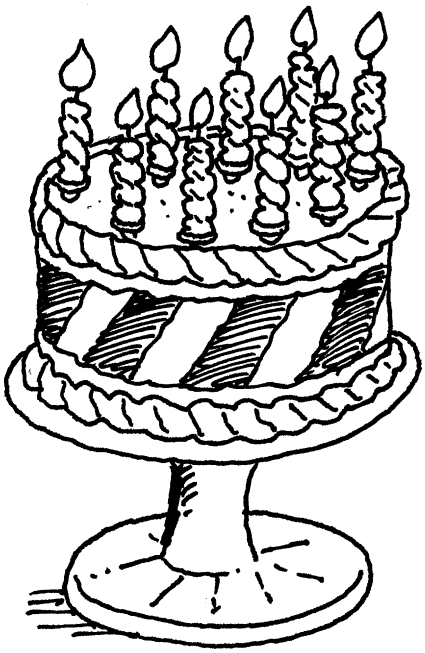
Birthdays

Mum and Paul are talking about birthdays.



They take Paul's age and double it.
Then they add 5.
The answer is 35.
Mum says this is her age.
How old is Paul?

Make up more problems like this.
Try to use some of these words:

double halve add subtract

21

Teaching objectives

Solve mathematical problems or puzzles.
Use known number facts to add mentally.
Carry out simple multiplication.

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Christmas tree

Rudolph put four stars on a tree.
He coloured each star either red or yellow.

In how many different ways can Rudolph colour the four stars?

Teaching objectives

Solve mathematical problems or puzzles.
Solve a problem by organising information.
Explain methods and reasoning.

22

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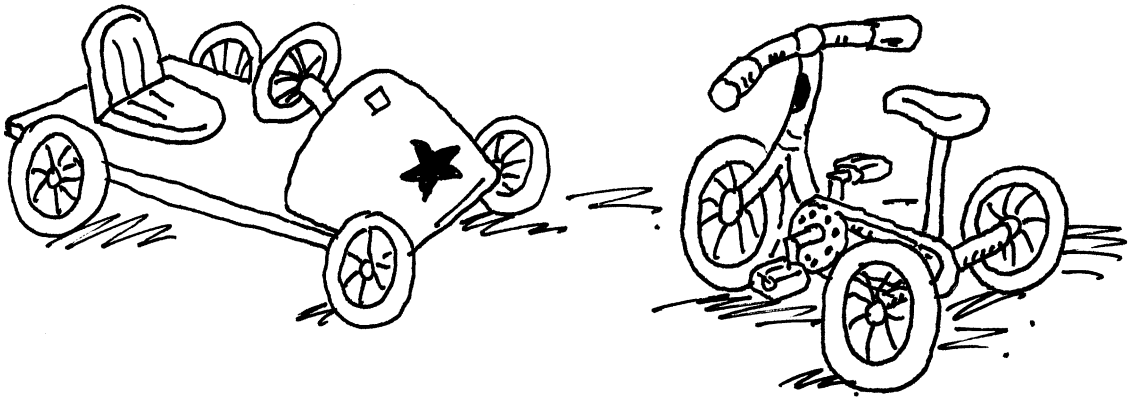
At the toy shop

The toy shop stocks tricycles and go-carts.
The tricycles have 3 wheels.
The go-carts have 5 wheels.



Suna counted the wheels.
He counted 37 altogether.

How many tricycles are there?
How many go-carts?

Find two ways to do it.

23

Teaching objectives

Solve mathematical problems or puzzles.
Recognise multiples of 3 and 5.
Add mentally a pair of two-digit numbers.

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Ben's numbers

Ben has written a list of different whole numbers.
The digits of each number add up to 5.
None of the digits is zero.

Here is one of Ben's numbers.

23

Ben has written all the numbers he can think of.
How many different numbers are there in his list?

Write all the numbers in order.

Teaching objectives

Solve a given problem by organising and interpreting data in a simple table.
Write whole numbers in figures; know what each digit represents.
Order whole numbers.

24

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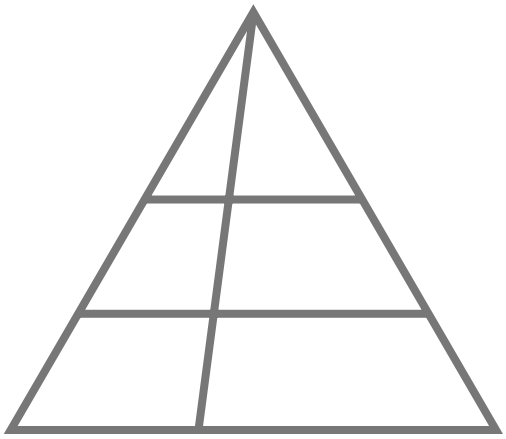
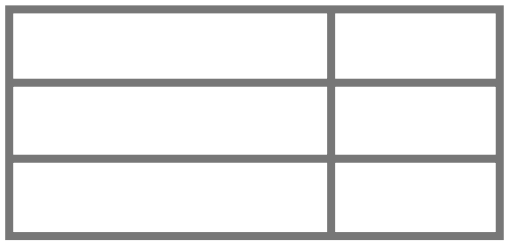
Spot the shapes 1

1. How many triangles can you count?



2. How many rectangles can you count?



3. Draw your own diagram to count triangles.
   How many can a friend find?
   Can you find more?

25

Teaching objectives

Solve mathematical problems or puzzles.
Visualise 2-D shapes.
Explain methods and reasoning.

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Puzzles and problems for Years 3 and 4

Rows of coins

1. Take five coins: 1p, 2p, 5p, 10p, 20p.
   Put them in a row using these clues.
   The total of the first three coins is 27p.
   The total of the last three coins is 31p.
   The last coin is double the value of the first coin.

2. Take six coins: two 1p, two 2p and two 5p.
   Put them in a row using these clues.
   Between the two 1p coins there is one coin.
   Between the two 2p coins there are two coins.
   Between the two 5p coins there are three coins.

   What if you take two 10p coins as well, and between them are four coins?

26

Teaching objectives

Solve word problems involving money.
Explain methods and reasoning.

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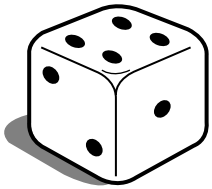
Roly poly

The dots on opposite faces of a dice add up to 7.

1. Imagine rolling one dice.

   The score is the total number of dots you can see.

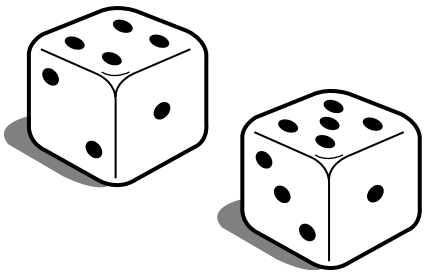
   You score 17.

   Which number is face down?

   How did you work out your answer?

2. Imagine rolling two dice.

   The dice do not touch each other.

   The score is the total number of dots you can see.

   Which numbers are face down to score 30?

Teaching objectives

Solve mathematical problems or puzzles.
Add three or four small numbers.
Explain methods and reasoning.

27

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Dan the detective

1. Dan the detective looked for a number.
   He found a two-digit number less than 50.
   The sum of its digits was 12.
   Their difference was 4.
   What number did Dan find?

2. Dan found a two-digit odd number.
   One of its digits was half the other.
   The number was greater than 50.
   What number did Dan find?

28

Teaching objectives

Solve a given problem by organising and interpreting data in a simple table.
Write whole numbers in figures; know what each digit represents.

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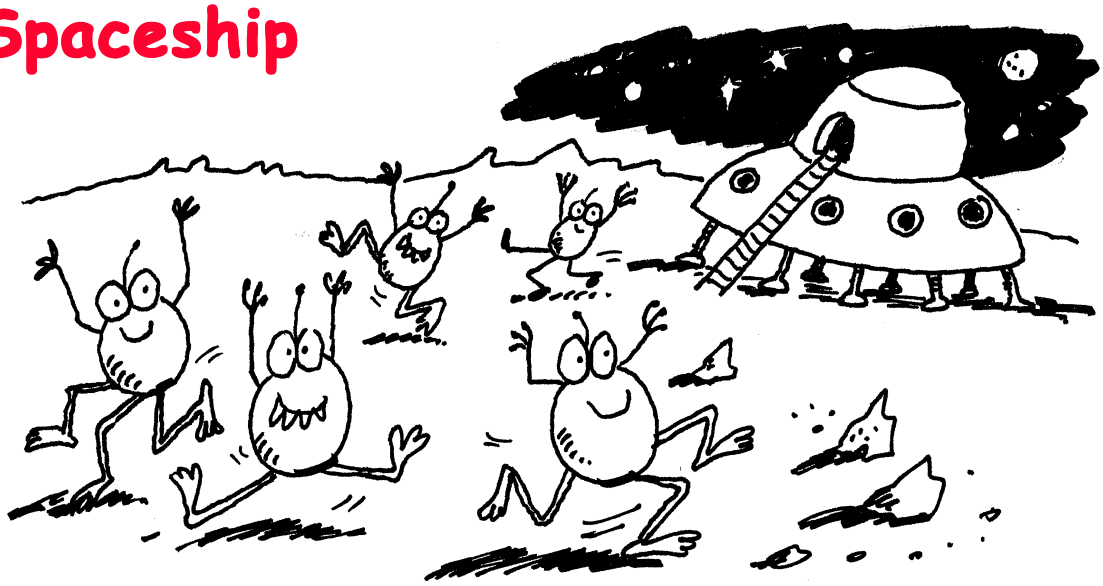
Spaceship



Some Tripods and Bipods flew from planet Zeno.
There were at least two of each of them.

Tripods have 3 legs.
Bipods have 2 legs.
There were 23 legs altogether.

How many Tripods were there?
How many Bipods?

Find two different answers.

Teaching objectives

Solve mathematical problems or puzzles.
Count on in steps of 2 or 3.
Know multiplication facts for 2 and 3 times tables.

29

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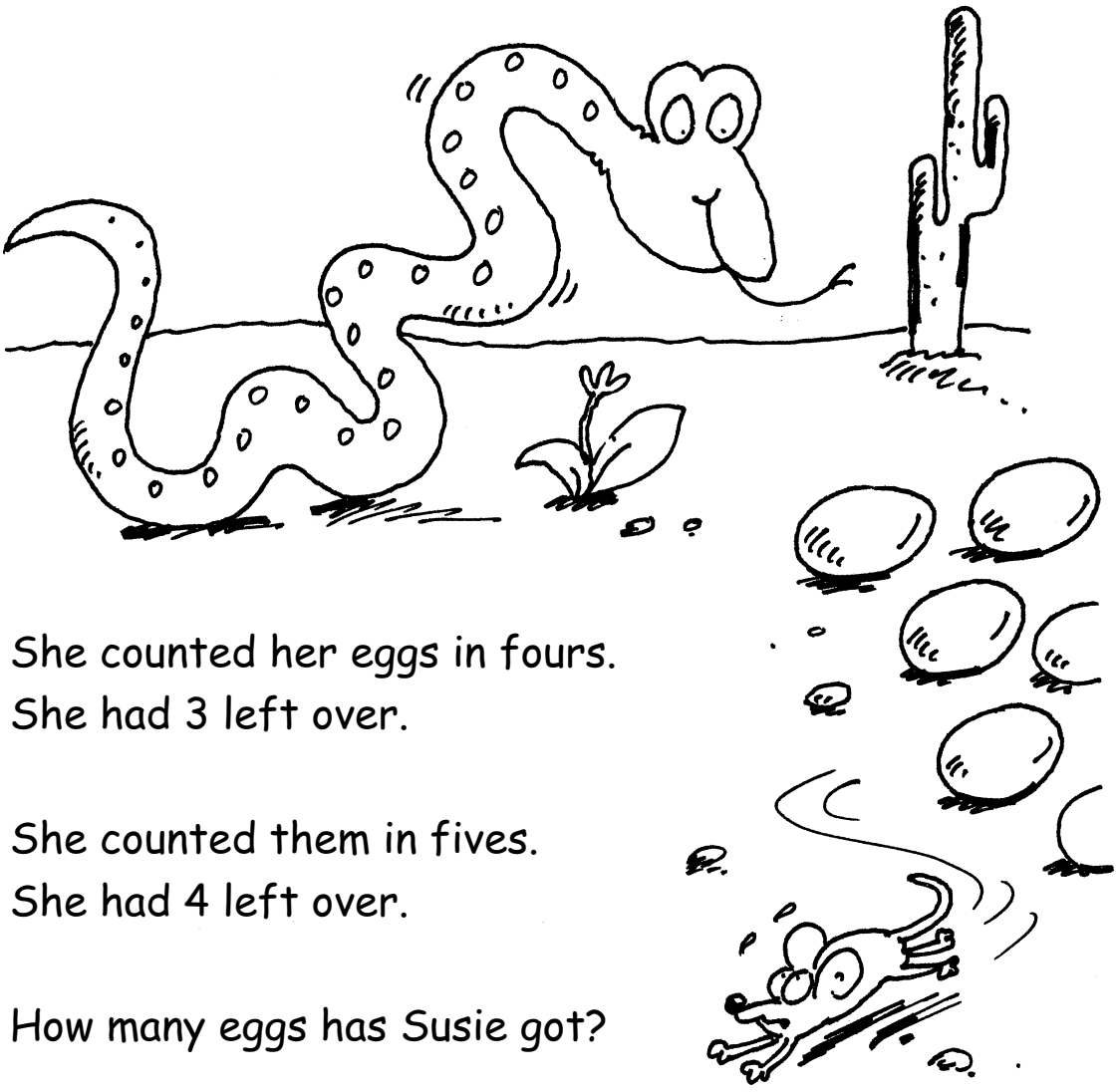
Susie the snake

Susie the snake has up to 20 eggs.



She counted her eggs in fours.
She had 3 left over.

She counted them in fives.
She had 4 left over.

How many eggs has Susie got?

30

Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts for 4 and 5 times tables.
Find remainders after division.

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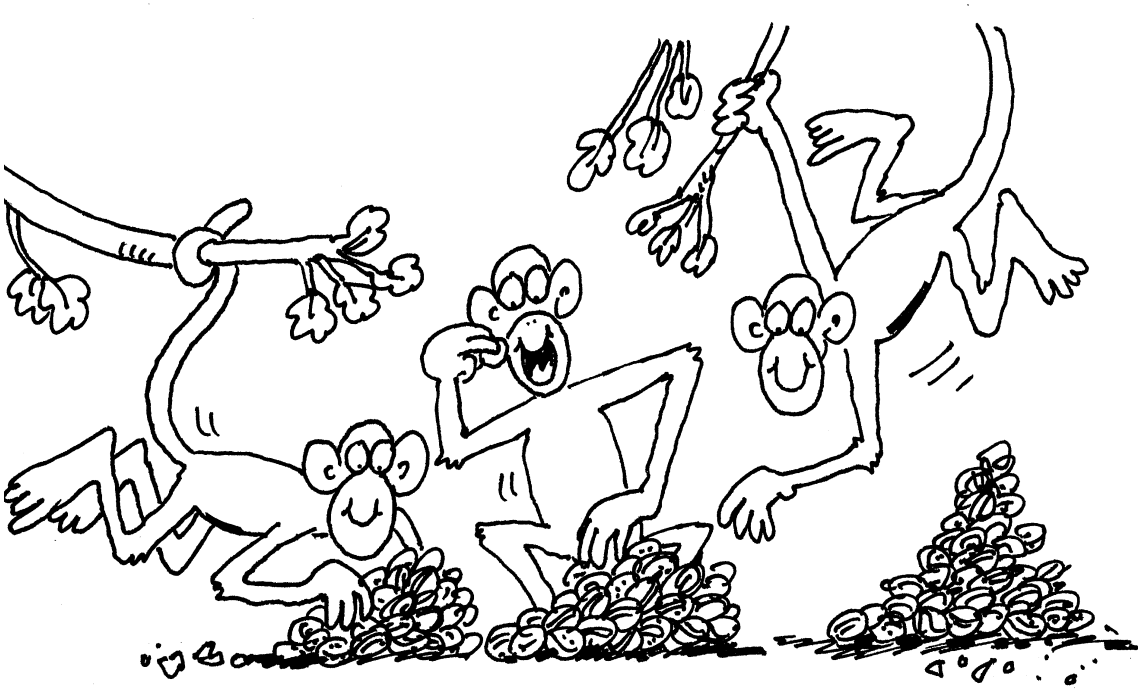
Three monkeys



Three monkeys ate a total of 25 nuts.
Each of them ate a different odd number of nuts.

How many nuts did each of the monkeys eat?
Find as many different ways to do it as you can.

Teaching objectives

Solve mathematical problems or puzzles.
Recognise odd and even numbers.
Add three or four small numbers mentally.

31

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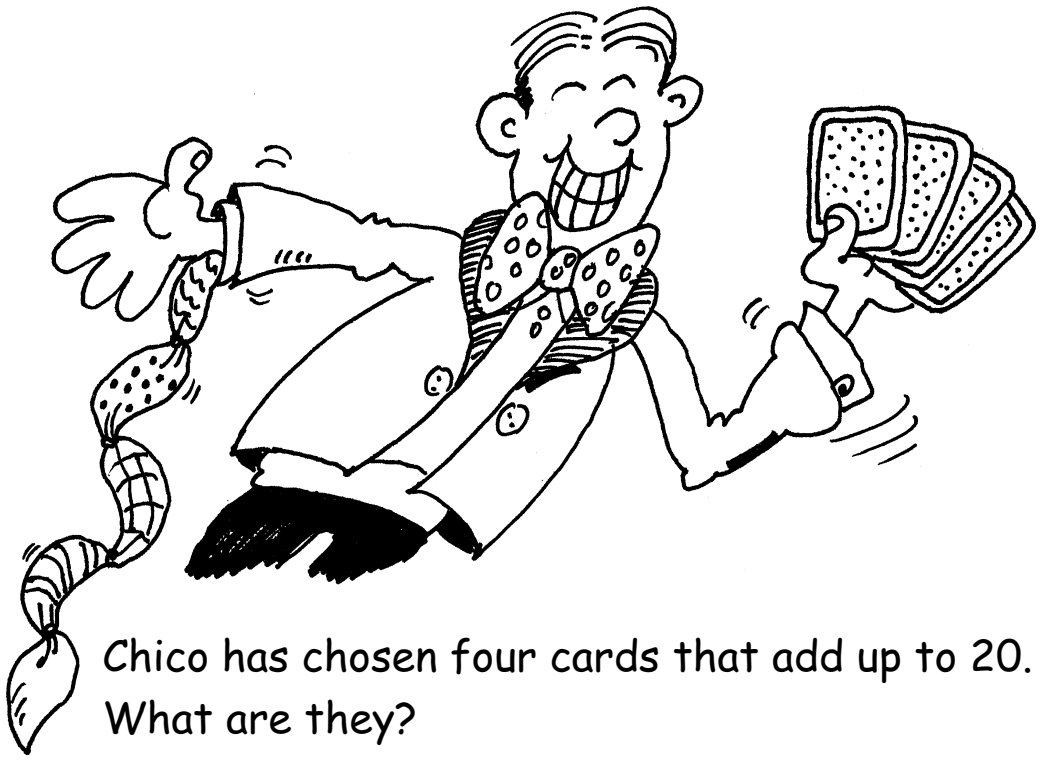
Card tricks

Chico's cards are all different.
There is a number from 1 to 8 on each card.



Chico has chosen four cards that add up to 20.
What are they?
There are seven different possibilities.
Try to find them all.

What if Chico has three cards that add up to 16?

32

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts up to 20.
Add three or four small numbers mentally.

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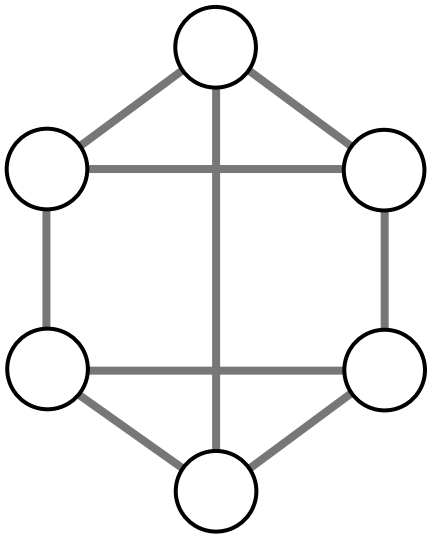
Neighbours

Use each of the numbers 1 to 6 once.
Write one in each circle.



Numbers next to each other must not be joined.
For example, 3 must not be joined to 2 or 4.

1 2 3 4 5 6

Teaching objectives

Solve mathematical problems or puzzles.
Order numbers 0 to 9.
Explain methods and reasoning.

33

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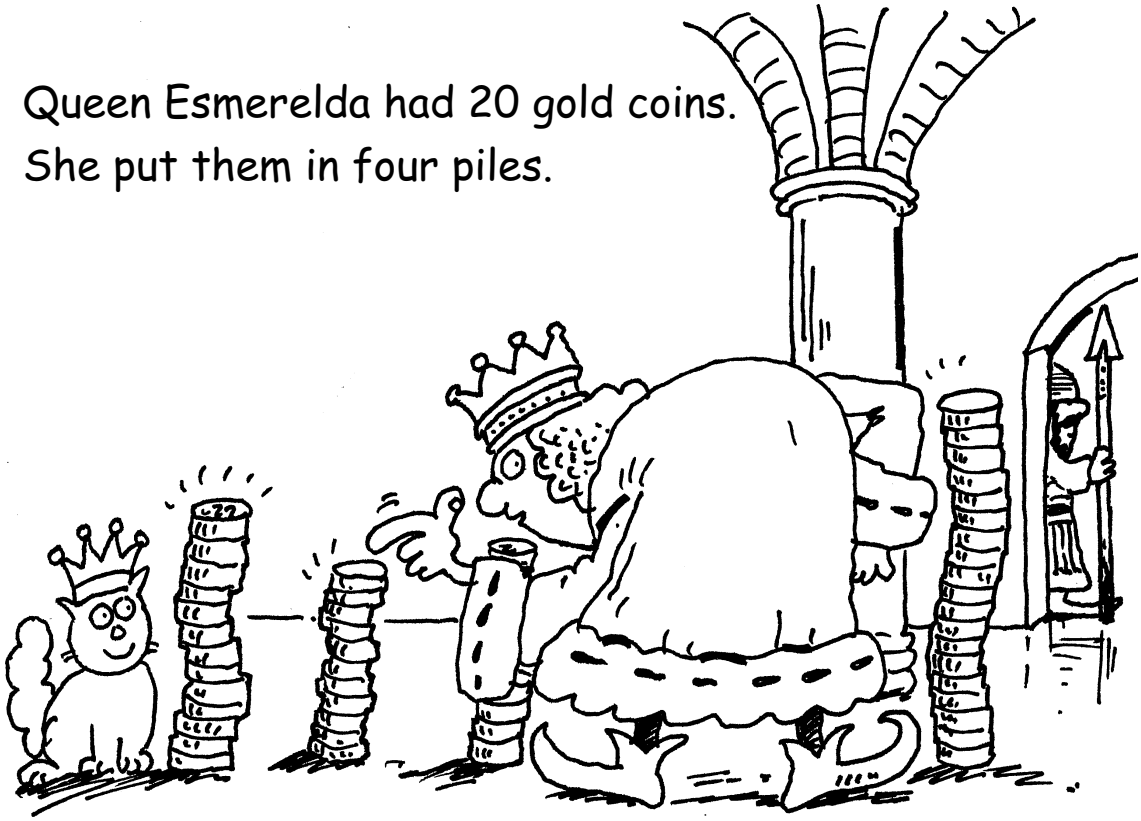
Queen Esmerelda's coins

Queen Esmerelda had 20 gold coins.
She put them in four piles.



- The first pile had four more coins than the second.
- The second pile had one less coin than the third.
- The fourth pile had twice as many coins as the second.

How many gold coins did Esmerelda put in each pile?

34

Teaching objectives

Solve mathematical problems or puzzles.
Use vocabulary of comparing and ordering numbers.
Explain methods and reasoning.

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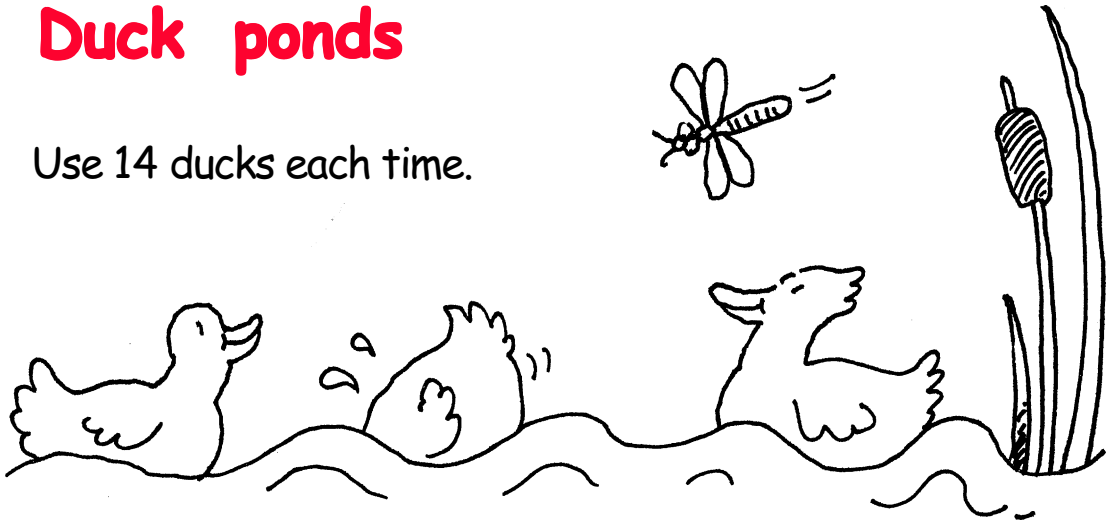
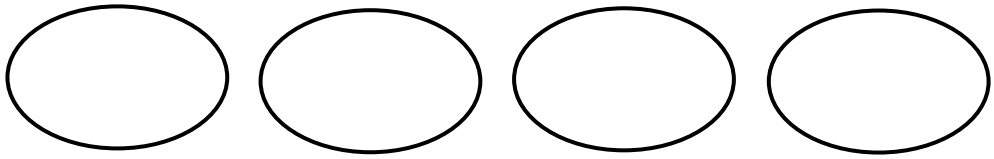
Duck ponds

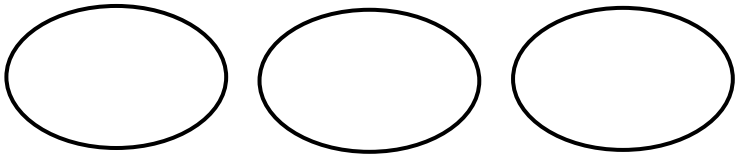
Use 14 ducks each time.



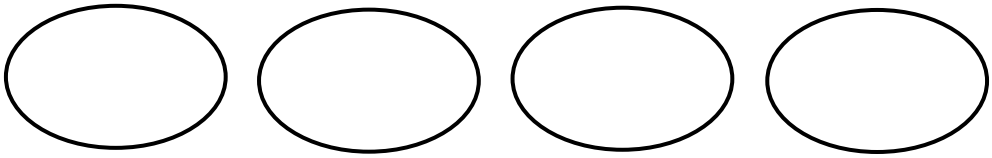
1. Make each pond hold two ducks or five ducks.



2. Make each pond hold twice as many ducks as the one before.



3. Make each pond hold one less duck than the one before.



Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts for 2 and 5 times tables.
Add three or four small numbers.

35

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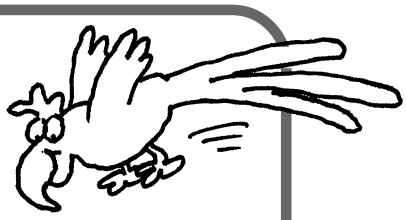
Treasure hunt



Jed and Jake are pirates.
Between them they have three precious jewels:
a ruby (R), a diamond (D) and an emerald (E).

Complete the table.
Show what jewels each pirate could have.

Jed	Ⓡ							
Jake	Ⓔ Ⓓ							

36

Teaching objectives

Solve a given problem by organising and interpreting data in a simple table.
Explain methods and reasoning.

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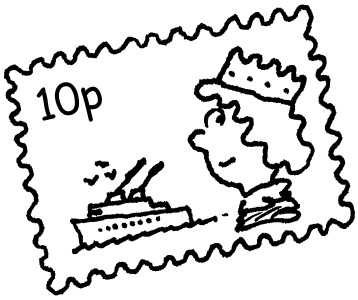
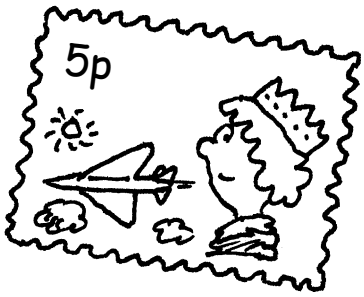
Stamps

Tilly's parcel cost 55p to post.

She stuck on eight stamps.
Each stamp was either 10p or 5p.





How many of each stamp did Tilly stick on her parcel?

Make up your own puzzle like this.
Ask a friend to do it.

Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts for 5 and 10 times tables.

37

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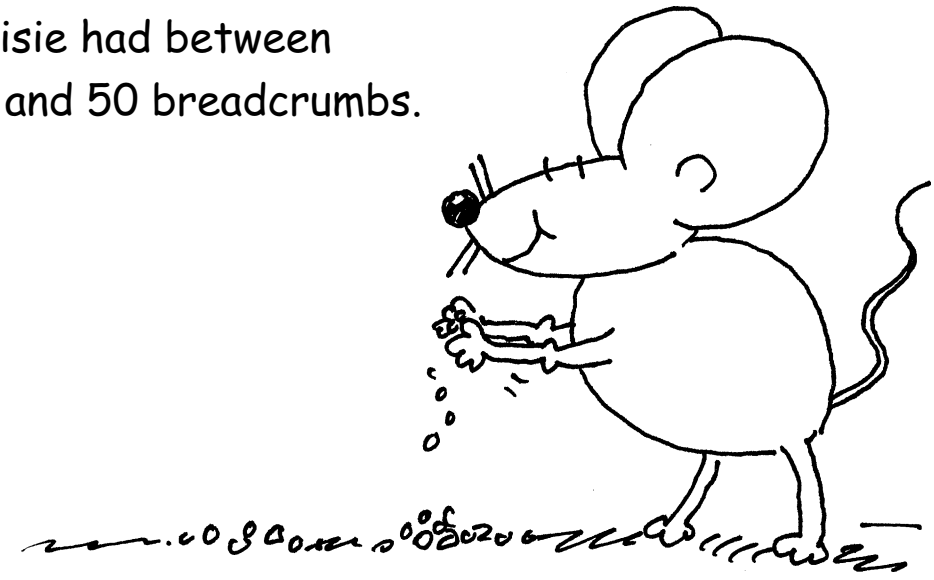
Maisie the mouse

Maisie had between
30 and 50 breadcrumbs.



She counted the breadcrumbs in fours.
There were 2 left over.

She counted them in fives.
There was 1 left over.

How many breadcrumbs did Maisie have?

38

Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts for 4 and 5 times tables.
Find remainders after division.

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Kieron's cats

Kieron has three cats.
Each is a different weight.

The first and second weigh 7 kg altogether.
The second and third weigh 8 kg altogether.
The first and third weigh 11 kg altogether.

What is the weight of each cat?

Teaching objectives

Solve mathematical problems or puzzles.
Know addition and subtraction facts to 20.
Explain methods and reasoning.

39

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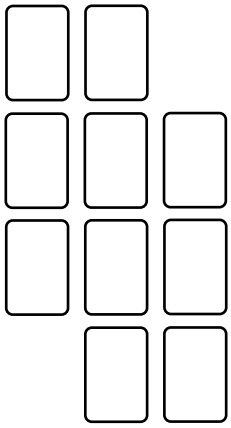
Next door numbers

Take ten cards numbered 0 to 9.

Arrange the cards like this.



Do it so that no two consecutive numbers are next to each other, horizontally, vertically or diagonally.

There are lots of ways to do it.
How many ways can you find?

40

Teaching objectives

Solve mathematical problems or puzzles.
Order numbers 0 to 9.
Explain methods and reasoning.

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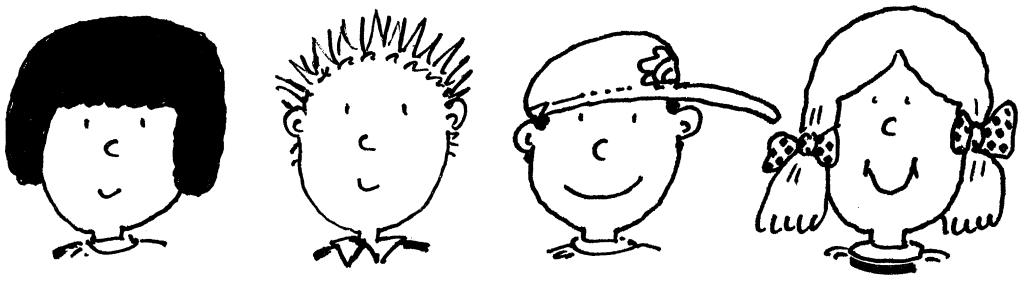
Nick-names

Dawn, Mark, Josh and Tina are friends.



They each have a nick-name.

Their nick-names are Spider, Curly, Ace and Fudgy, but not in that order.

What is the nick-name of each of the friends?

Clues

- Josh plays tennis with Curly and goes swimming with Ace.
- Tina has been on holiday with Curly but travels to school with Fudgy.
- Spider, Curly and Dawn play in the football team.
- Spider sometimes goes to tea with Josh.

Teaching objectives

Solve mathematical problems or puzzles.
Solve a problem by organising information in a table.
Explain methods and reasoning.

41

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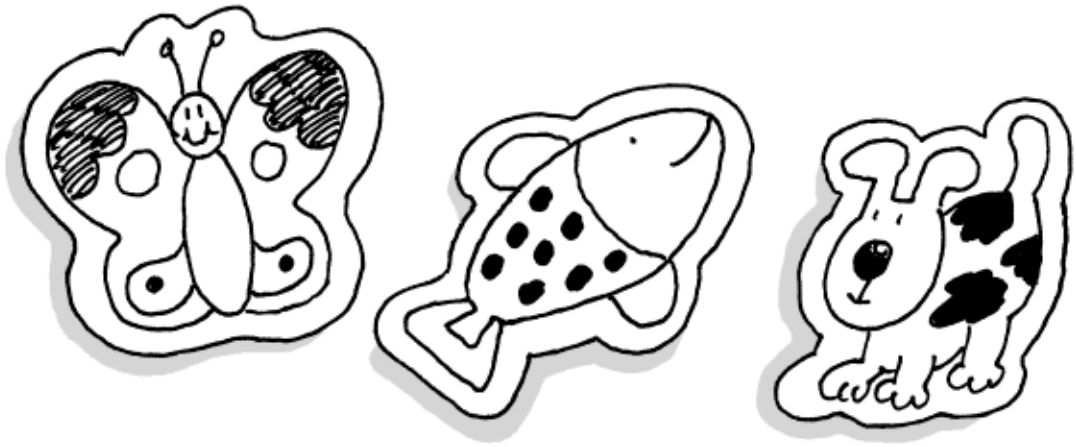
Stickers

The twins collected some animal stickers.
They each had the same total number.



Winston had 3 full sheets and 4 loose stickers.
Wendy had 2 full sheets and 12 loose stickers.

Every full sheet has the same number of stickers.
How many stickers are there in a full sheet?

42

Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts.
Explain methods and reasoning.

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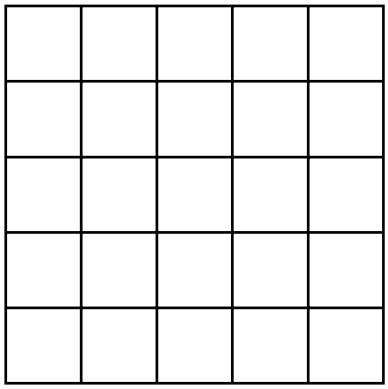
Odds and evens

You need 13 counters or coins.



Draw a 5 by 5 grid.
Put counters on it.
You can put only one counter in each space.

1. Place 13 counters.
   Get an **odd** number of them in each row and column and the two main diagonals.

2. Place 10 counters.
   Get an **even** number of them in each row and column and the two main diagonals.

Teaching objectives

Solve mathematical problems or puzzles.
Recognise odd and even numbers.
Explain methods and reasoning.

43

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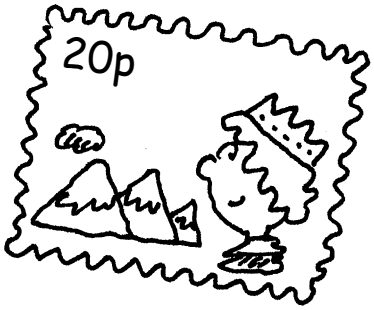
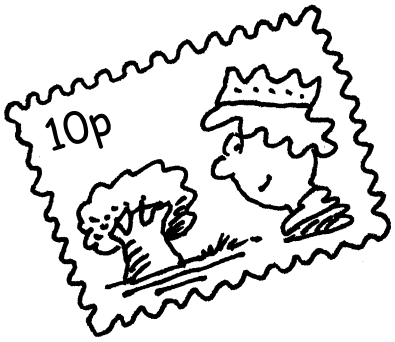
More stamps

Rosie spent £2 on 10p and 20p stamps.





She bought three times as many
10p stamps as 20p stamps.

How many of each stamp did she buy?

44

Teaching objectives

Solve mathematical problems or puzzles.
Begin to use ideas of simple ratio and proportion.
Explain methods and reasoning.

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Sandcastles

Lisa went on holiday.

In 5 days she made 80 sandcastles.

Each day she made 4 fewer castles than the day before.

How many castles did she make each day?

Lisa went on making 4 fewer castles each day.

How many castles did she make altogether?

Teaching objectives

Solve mathematical problems or puzzles.
Add two-digit numbers.

45

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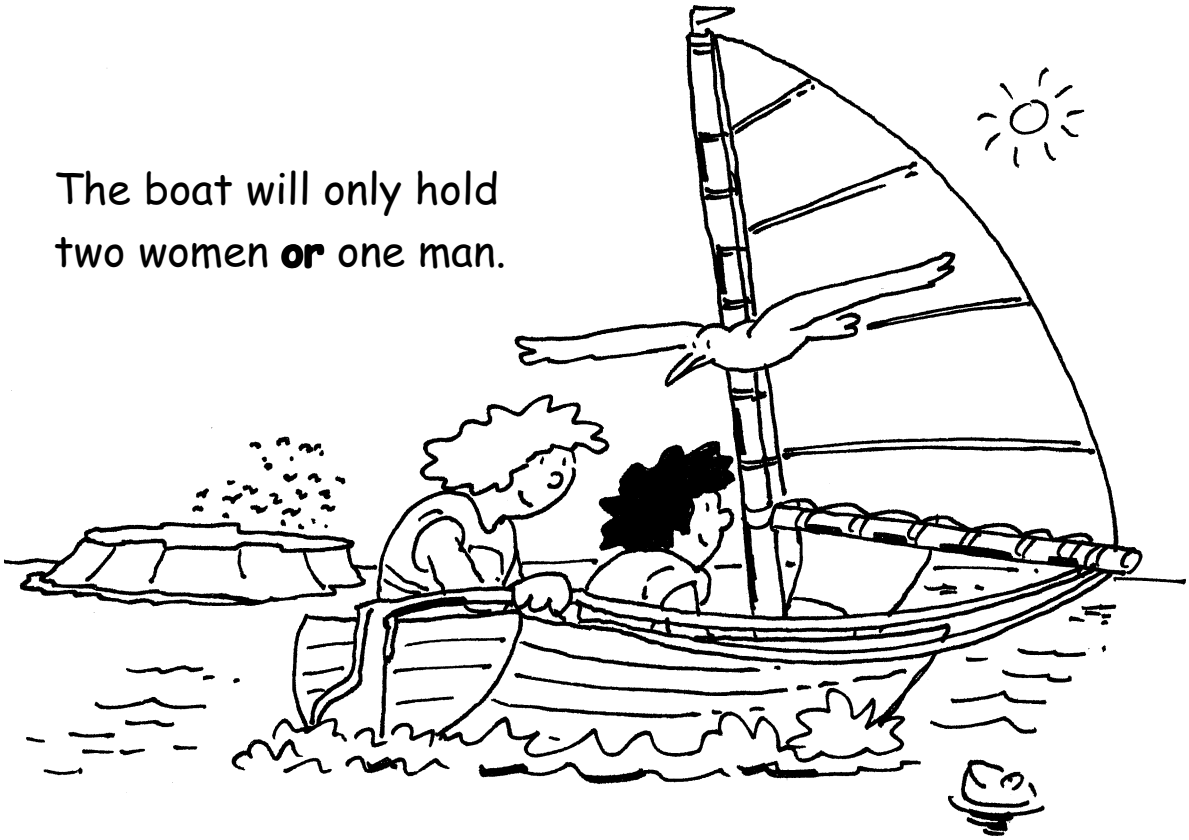
Sail away

Two men and two women want to sail to an island.

The boat will only hold two women **or** one man.



How can all four of them get to the island?

46

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

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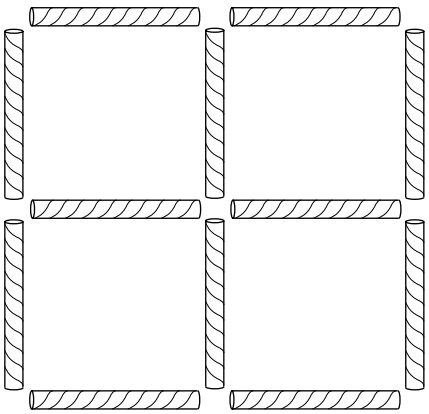
Straw squares

You need 20 straws all the same length.

There are 12 straws in this pattern of 5 squares.



Take 20 straws.
Arrange them to make as many squares as you can.
Don't bend or break the straws!

How many squares did you make?

Teaching objectives

Solve mathematical problems or puzzles.
Visualise 2-D shapes.

47

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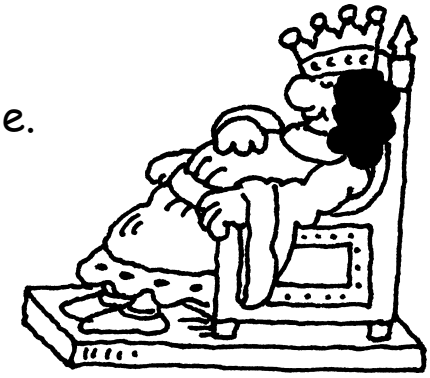
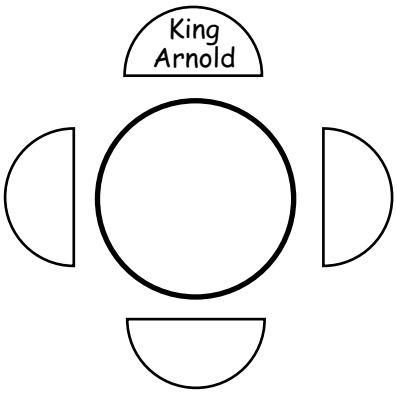
King Arnold

King Arnold sits at a Round Table.



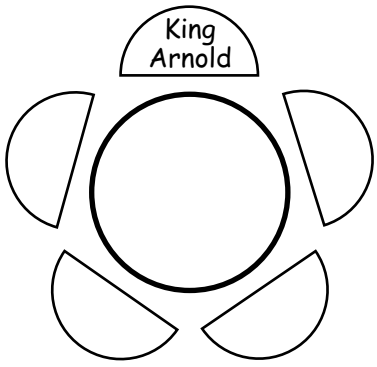
There are 3 empty seats.

In how many different ways can 3 knights sit in them?



What if there are 4 empty seats?

In how many different ways can 4 knights sit in them?



48

Teaching objectives

Solve mathematical problems or puzzles.
Solve a problem by organising information.
Explain methods and reasoning.

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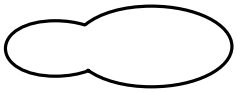
Footsteps in the snow

Little has size 2 boots.

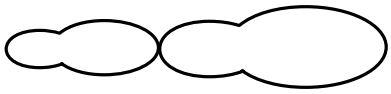
Middle has size 3 boots.

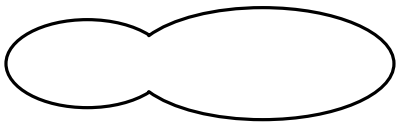
They are one and a half times the length of Little's boots.



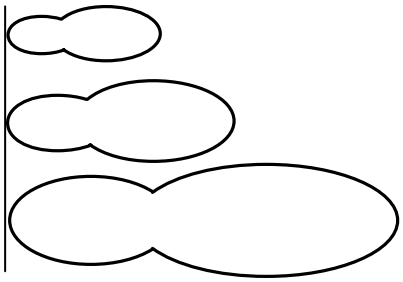
Big has size 5 boots.

A little boot and a middle boot are the same length as a big boot.





They start with the heels of their boots on the same line.



They each walk heel to toe.

When will all three heels be in line again?

Teaching objectives

Solve mathematical problems or puzzles.
Recognise multiples of 2, 3 and 5.

49

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Ski lift

On a ski lift the chairs are equally spaced.
They are numbered in order from 1.

Kelly went skiing.
She got in chair 10 to go to the top of the slopes.

Exactly half way to the top, she passed chair 100 on its way down.

How many chairs are there on the ski lift?

Make up more problems like this.

50

Teaching objectives

Solve mathematical problems or puzzles.
Solve a problem by organising information.
Explain methods and reasoning.

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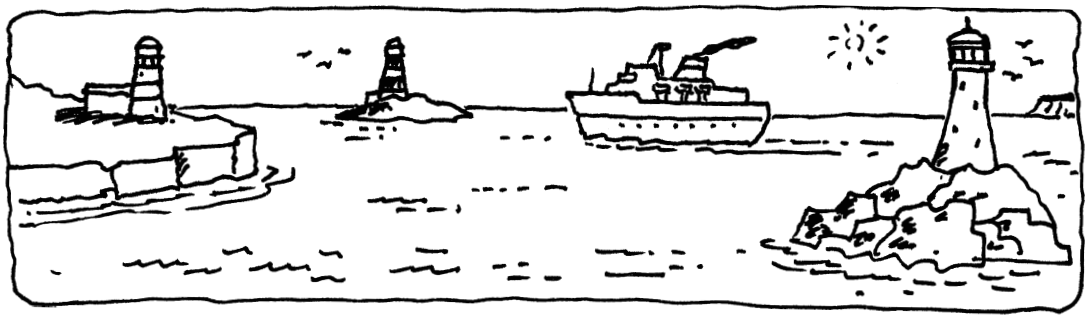
Lighthouses

On the coast there are three lighthouses.



The first light shines for 3 seconds, then is off for 3 seconds.

The second light shines for 4 seconds, then is off for 4 seconds.

The third light shines for 5 seconds, then is off for 5 seconds.

All three lights have just come on together.

When is the first time that all three lights will be off?

When is the next time that all three lights will come on at the same moment?

Teaching objectives

Solve mathematical problems or puzzles.
Recognise multiples of 6, 8 and 10.
Explain methods and reasoning.

51

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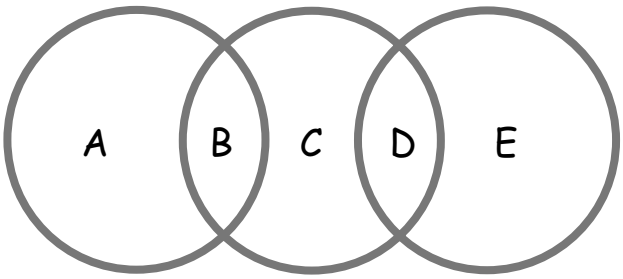
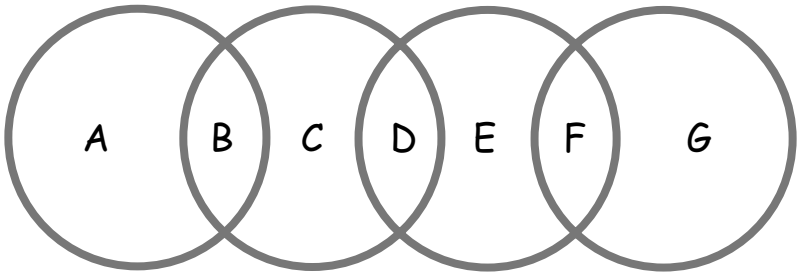
Circle sums

1. Use each of the digits 1 to 5 once.
   Replace each letter by one of the digits.
   Make the total in each circle the same.



2. Now use each of the digits 1 to 7 once.
   Make the total in each circle the same.



3. What if you used five circles and the digits 1 to 9?

52

Teaching objectives

Solve mathematical problems or puzzles.
Add several single digits.
Know addition and subtraction facts to 20.

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Puzzles and problems for Years 5 and 6

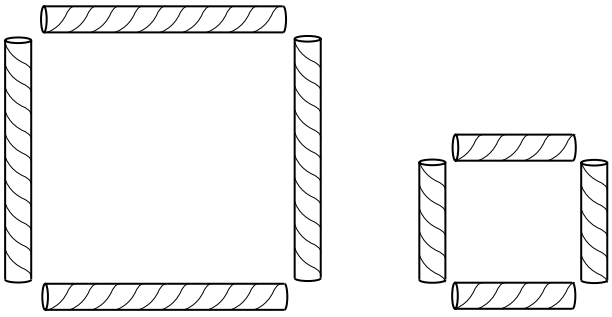
Square it up

You need six drinking straws each the same length.

Cut two of them in half.

You now have eight straws, four long and four short.

You can make 2 squares from the eight straws.



Arrange your eight straws to make 3 squares, all the same size.

53

Teaching objectives

Solve mathematical problems or puzzles.
Visualise 2-D shapes.

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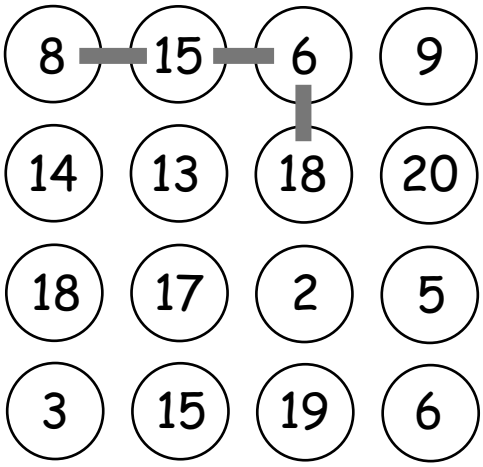
Joins

Join any four numbers.
Find their total.
Joins can go up, down or sideways, but not diagonally.
The score shown is 8 + 15 + 6 + 18 = 47.



Find the highest possible score.
Find the lowest possible score.

Try joining five numbers.
Now try joining five numbers using only diagonal joins.

Teaching objectives
Solve mathematical problems or puzzles.
Add and subtract two-digit numbers mentally.

54

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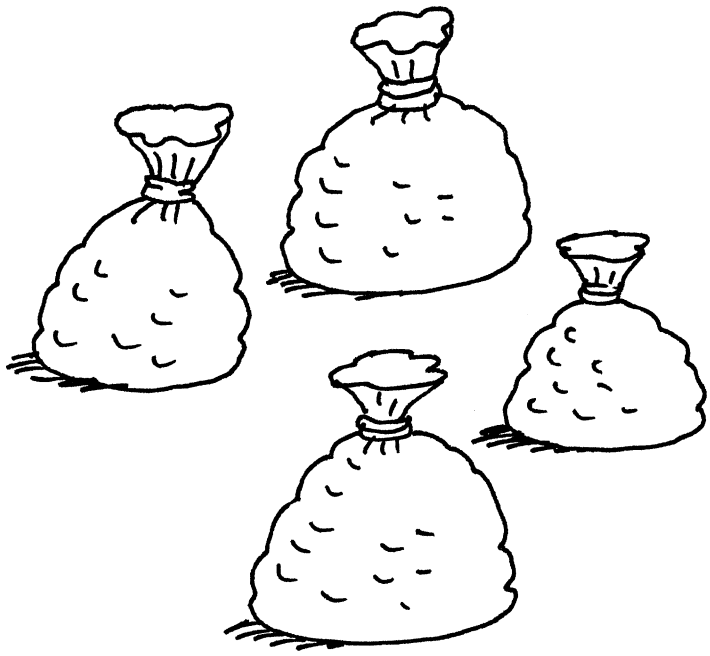
Money bags

Ram divided 15 pennies among four small bags.

He could then pay any sum of money from 1p to 15p, without opening any bag.

How many pennies did Ram put in each bag?



55

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

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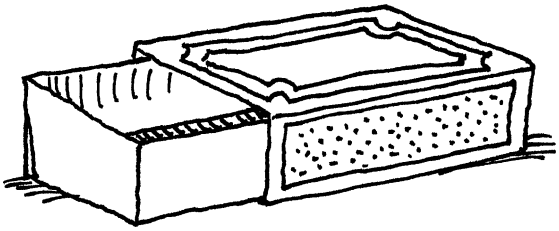
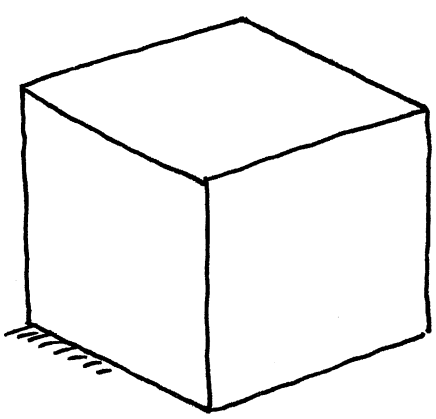
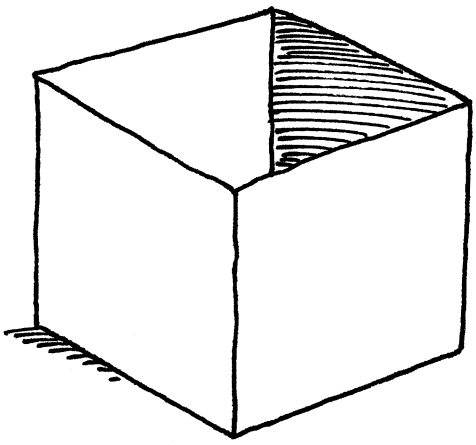
A perfect match

1. A matchbox tray slides into its outer cover.
   In how many different ways can you do this?



2. Imagine a cube and an open box just large enough to hold it.
   In how many different ways can you fit the cube into the box?





Teaching objectives

Solve mathematical problems or puzzles.
Visualise 3-D shapes.

56

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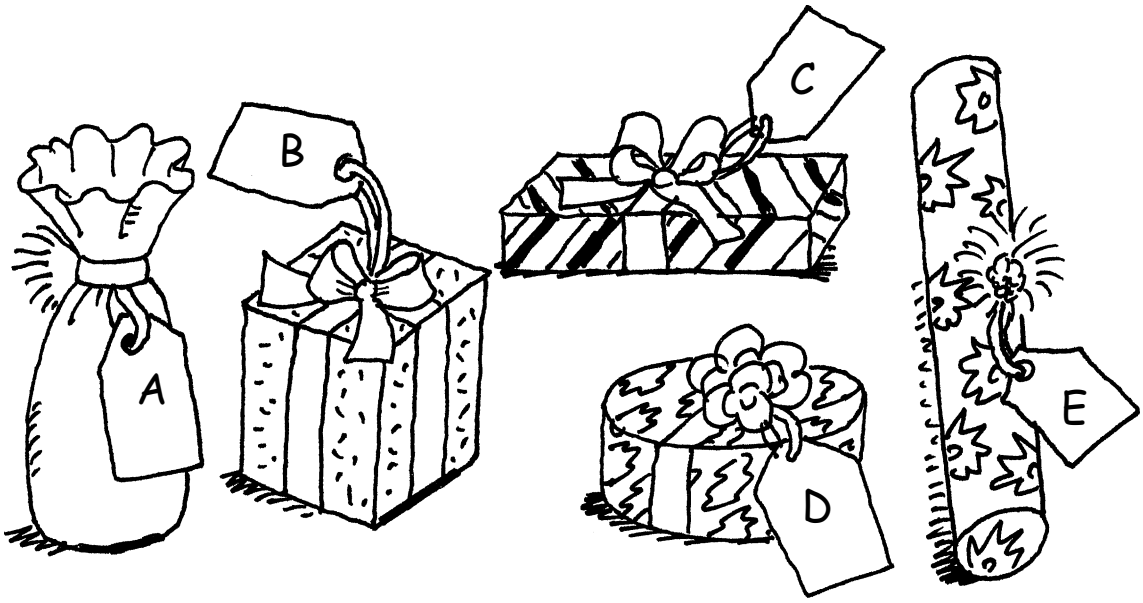
Presents

Gurmit paid £21 for five presents.



For A and B he paid a total of £6.
For B and C he paid a total of £10.
For C and D he paid a total of £7.
For D and E he paid a total of £9.

How much did Gurmit pay for each present?

57

Teaching objectives

Solve a given problem by organising information.
Explain methods and reasoning.

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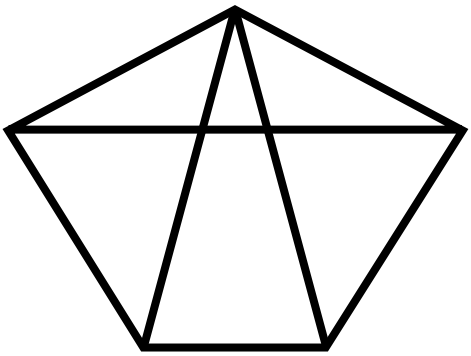
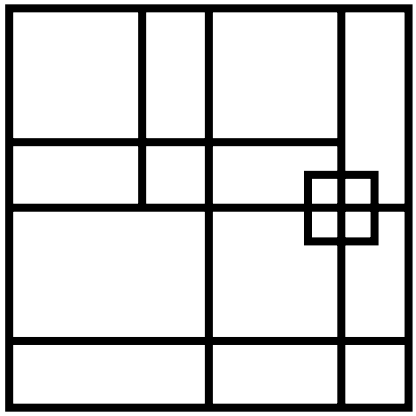
Spot the shapes 2

1. How many triangles can you count?



2. How many squares can you count?



3. Draw your own diagram to count triangles.
   Don't use too many lines!
   How many triangles can a friend find?
   Can you find more?

Teaching objectives

Solve mathematical problems or puzzles.
Visualise 2-D shapes.
Explain methods and reasoning.

58

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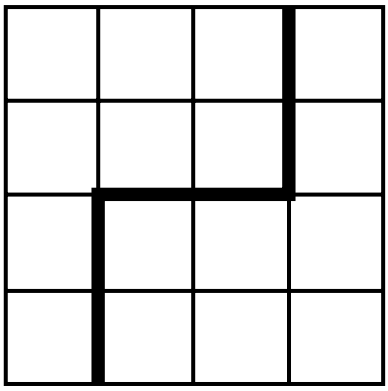
Four by four

You need some squared paper.

This 4 by 4 grid is divided into two identical parts.
Each part has the same area and the same shape.



Find five more ways of dividing the grid into two identical parts by drawing along the lines of the grid.
Rotations and reflections do not count as different!

Explore ways of dividing a 4 by 4 grid into two parts with equal areas but different shapes.

59

Teaching objectives

Solve mathematical problems or puzzles.
Visualise 2-D shapes.
Find fractions of shapes.

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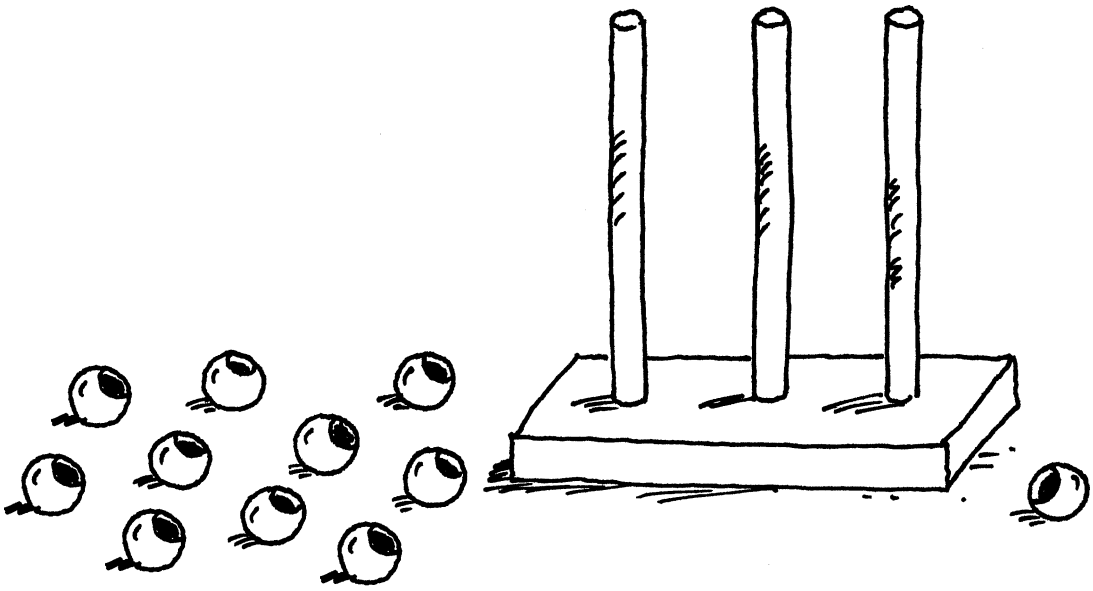
Three digits

Imagine you have 25 beads.
You have to make a three-digit number on an abacus.
You must use all 25 beads for each number you make.



How many different three-digit numbers can you make?
Write them in order.

Teaching objectives

Solve mathematical problems or puzzles.
Know what each digit represents.
Order a set of whole numbers.

60

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Make five numbers

Take ten cards numbered 0 to 9.

Each time use all ten cards.

Arrange the cards to make:

a. five numbers that are multiples of 3

b. five numbers that are multiples of 7

c. five prime numbers

Make up more problems to use all ten cards to make five special numbers.

61

Teaching objectives

Solve mathematical problems or puzzles.
Know 3 and 7 times tables.
Recognise prime numbers.

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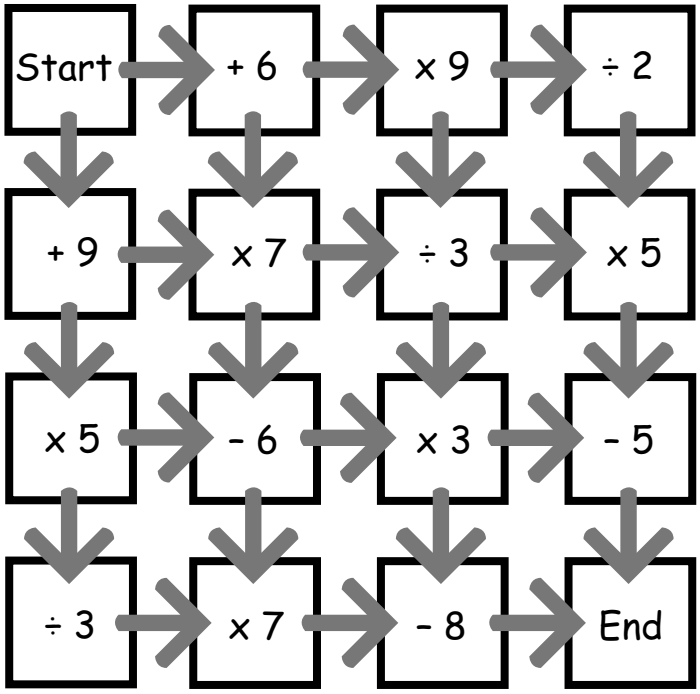
Maze

Start with zero.

Find a route from 'Start' to 'End' that totals 100 exactly.



Which route has the highest total?

Which has the lowest total?

Now try some different starting numbers.

Teaching objectives

Solve mathematical problems or puzzles.
Add and subtract two-digit numbers mentally.
Multiply and divide by single-digit numbers.

62

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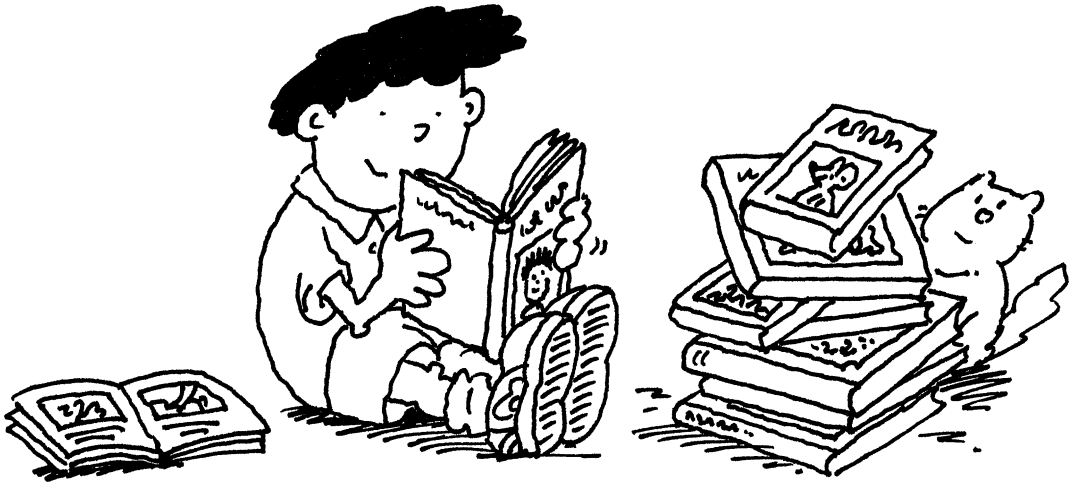
Jack's book

The pages of Jack's book are numbered from 1.



The page numbers have a total of 555 digits.

How many pages has the book?

How many of the digits are a 5?

63

Teaching objectives

Solve mathematical problems or puzzles.
Know what each digit represents.

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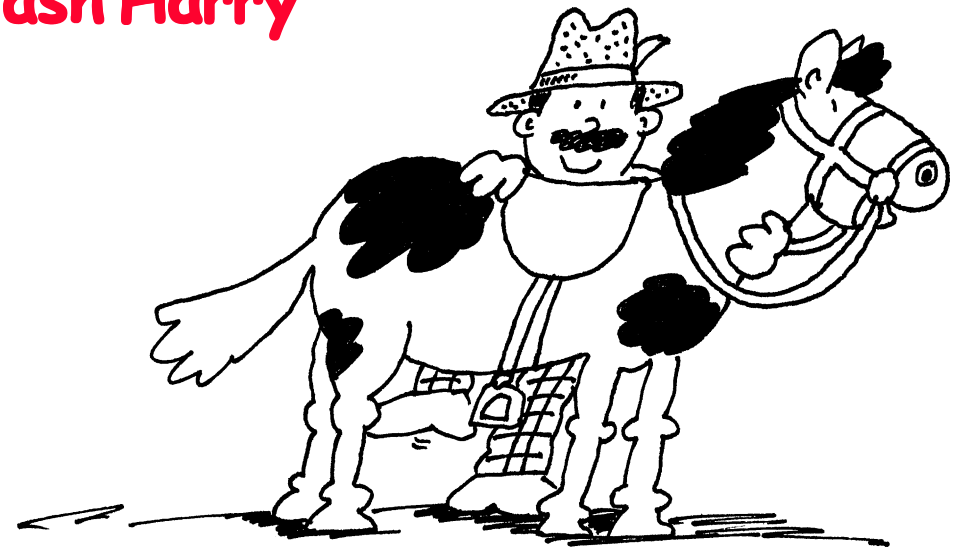
Flash Harry



In April Flash Harry bought a saddle for £100.
In May he sold it for £200.

In June he was sorry he had sold it.
So he bought it back for £300.

In July he got tired of it.
So he sold it for £400.

Overall, did Flash Harry make or lose money?
How much did he make or lose?

Teaching objectives

Solve mathematical problems or puzzles.
Use negative numbers.

64

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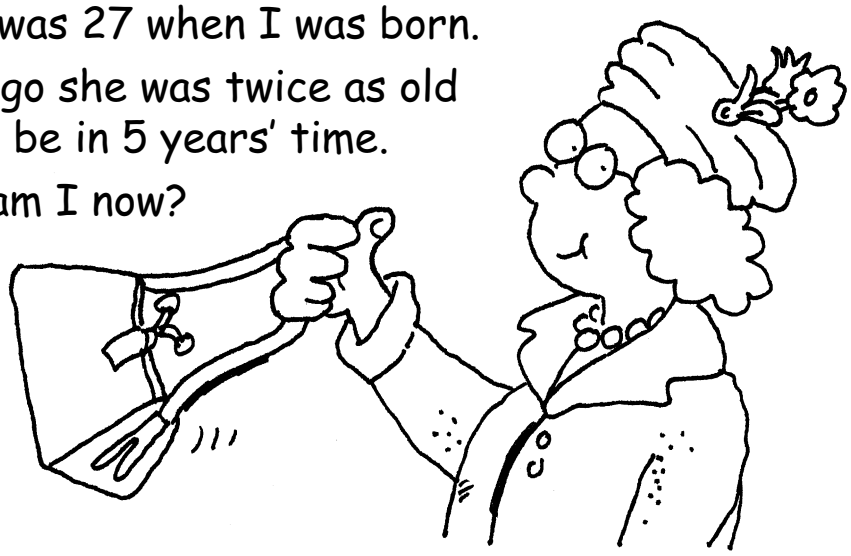
Age old problems

1. My age this year is a multiple of 8.
   Next year it will be a multiple of 7.
   How old am I?

2. Last year my age was a square number.
   Next year it will be a cube number.
   How old am I?
   How long must I wait until my age is both a square number and a cube?

3. My Mum was 27 when I was born.
   8 years ago she was twice as old as I shall be in 5 years' time.
   How old am I now?



65

Teaching objectives

Solve mathematical problems or puzzles.
Know multiplication facts to 10 x 10.
Recognise square and cube numbers.

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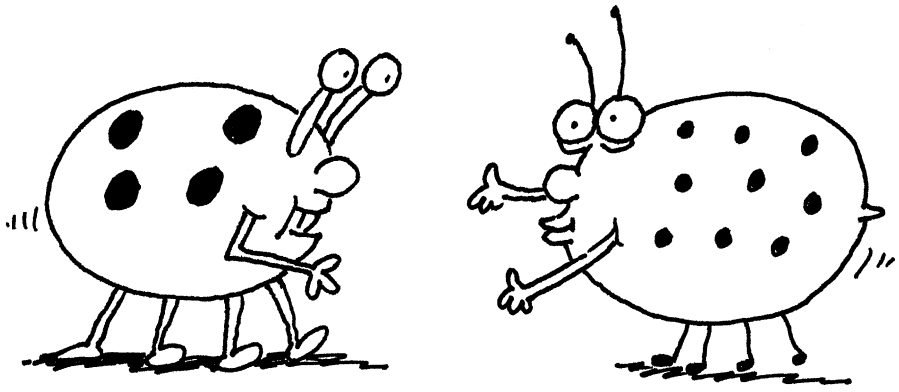
Zids and Zods



Zids have 4 spots.
Zods have 9 spots.

Altogether some Zids and Zods have 48 spots.
How many Zids are there?
How many Zods?

What if Zids have 5 spots, Zods have 7 spots, and there are 140 spots altogether?
Find as many solutions as you can.

Teaching objectives
Solve mathematical problems or puzzles.
Know multiplication facts to 10 x 10.
Add two-digit numbers mentally.

66

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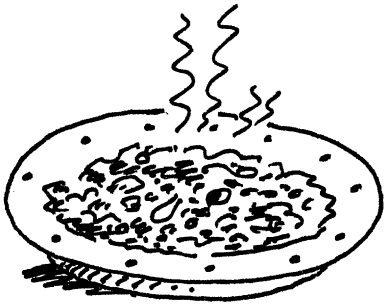
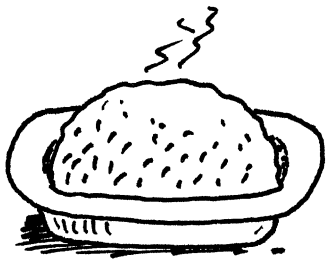
Franco's fast food





This is what food costs at Franco's café.

1 curry and 1 tea cost £4.
2 curries and 2 puddings cost £9.
1 pudding and 2 teas cost £2.

What do you have to pay in total for
1 curry, 1 pudding and 1 tea?
What does each item cost on its own?

67

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

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Albert Square

36 people live in the eight houses in Albert Square.
Each house has a different number of people living in it.
Each line of three houses has 15 people living in it.
How many people live in each house?

Teaching objectives

Solve mathematical problems or puzzles.
Add several small numbers mentally.
Explain methods and reasoning.

68

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Coins on the table

Anna put some 10p coins on the table.
One half of them were tails up.

Anna turned over two of the coins, and then one third of them were tails up.

How many coins did Anna put on the table?

69

Teaching objectives

Solve mathematical problems or puzzles.
Understand simple fractions.
Explain methods and reasoning.

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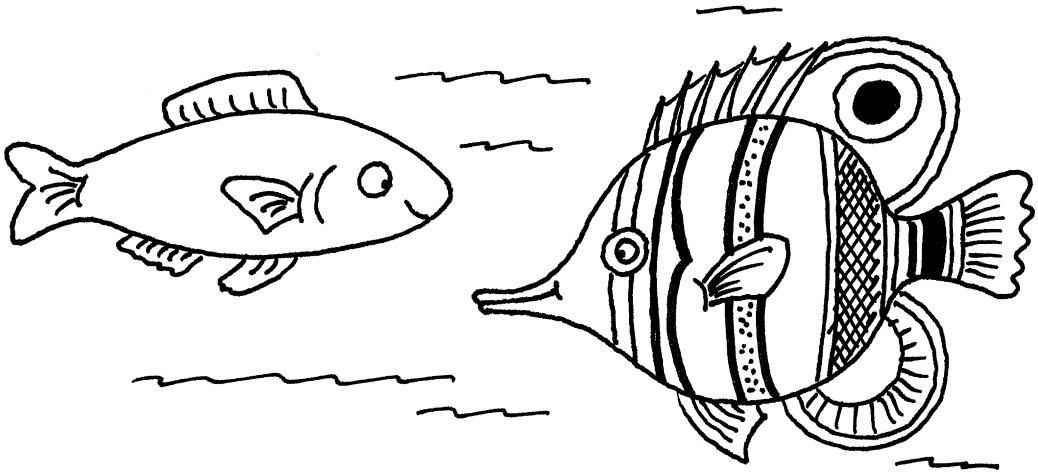
A bit fishy

A goldfish costs £1.80.
An angel fish costs £1.40.



Nasreen paid exactly £20 for some fish.
How many of each kind did she buy?

Teaching objectives

Solve problems involving ratio and proportion.
Choose and use efficient calculation strategies to solve a problem.
Explain methods and reasoning.

70

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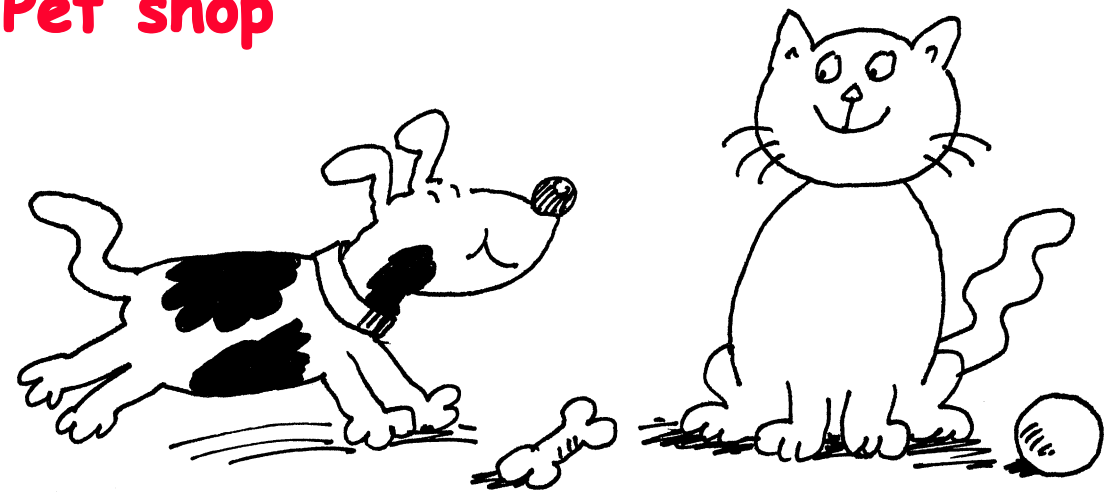
Pet shop



1. Jim bought a cat and dog for £60 each.
   Later he sold them.
   He made a profit of 20% on the dog.
   He made a loss of 20% on the cat.
   How much did he get altogether when he sold the cat and dog?

2. Jim bought another cat and dog.
   He sold them for £60 each.
   He made a profit of 20% on the dog.
   He made a loss of 20% on the cat.
   Did he make a profit or loss on the whole deal?

71

Teaching objectives

Solve mathematical problems or puzzles.
Find simple percentages.

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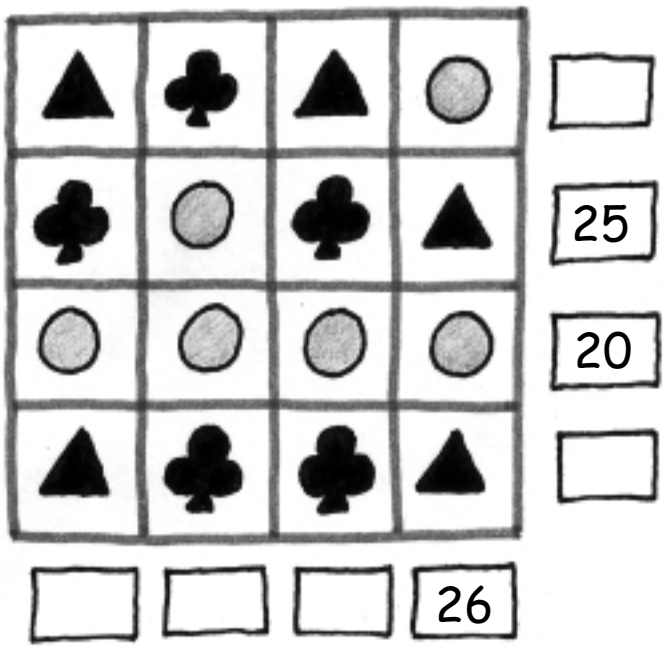
Shape puzzle

Each shape stands for a number.

The numbers shown are the totals of the line of four numbers in the row or column.



Find the remaining totals.

Teaching objectives

Solve mathematical problems or puzzles.
Use a symbol to stand for an unknown number.
Explain methods and reasoning.

72

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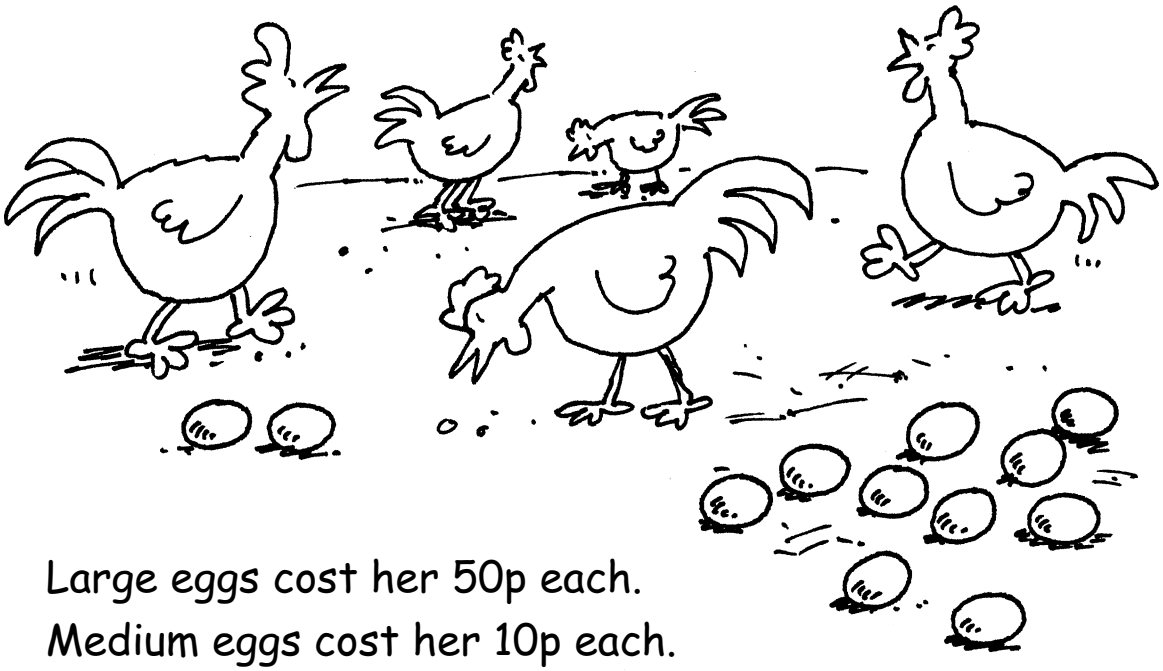
Eggs

Mrs Choy spent exactly £10 on 100 eggs for her shop.



Large eggs cost her 50p each.
Medium eggs cost her 10p each.
Small eggs cost her 5p each.

For two of the sizes, she bought the same number of eggs.
How many of each size did she buy?

73

Teaching objectives

Solve problems involving ratio and proportion.
Explain methods and reasoning.

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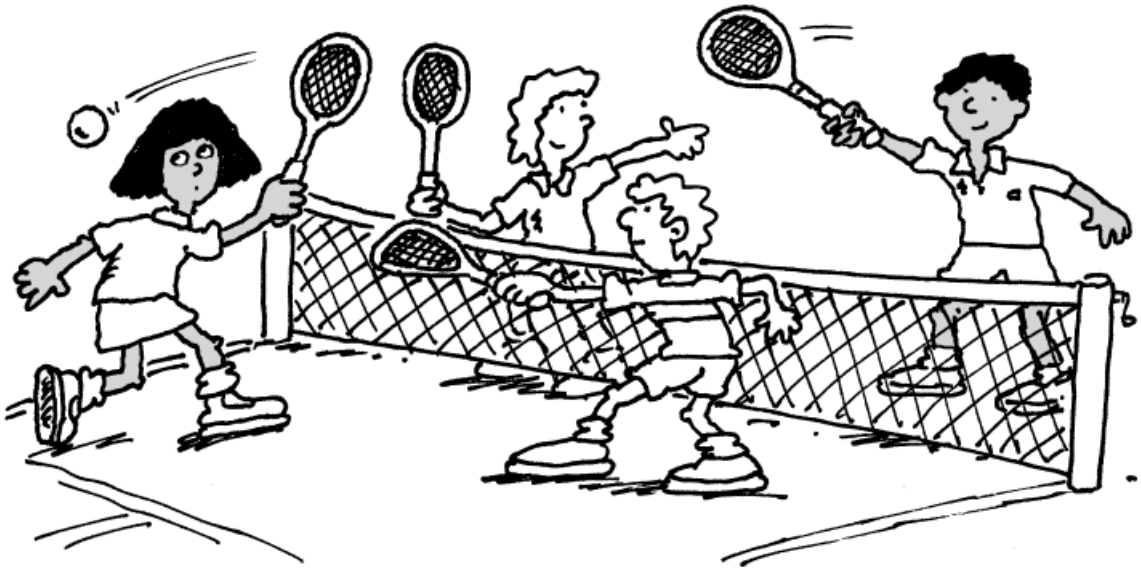
Anyone for tennis?

Two boys and two girls can play tennis.



Ali said: 'I will only play if Holly plays.'
Holly said: 'I won't play if Ben is playing.'
Ben said: 'I won't play if Luke or Laura plays.'
Luke said: 'I will only play if Zoe plays.'
Zoe said: 'I don't mind who I play with.'

Which two boys and which two girls play tennis?

Teaching objectives
Solve a problem by extracting and interpreting data.
Explain methods and reasoning.

74

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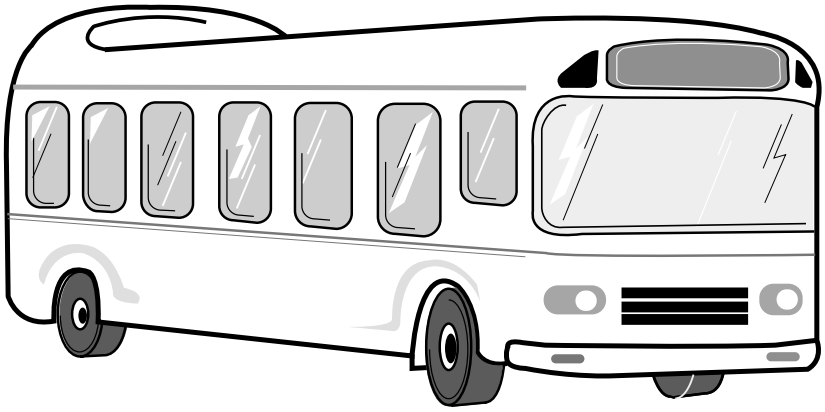
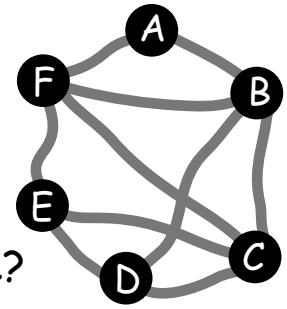
Bus routes





Six towns are connected by bus routes.
The bus goes from A back to A.
It visits each of the other towns once.
How many different bus routes are there?

This table shows the bus fare for each direct route.
B to A costs the same as A to B, and so on.

A to B	B to C	C to D	D to E	E to F	F to A	B to D	B to F	C to E	C to F
£4	£3	£4	£4	£3	£4	£5	£3	£2	£2

Which round trip from A to A is the cheapest?

75

Teaching objectives

Solve a problem by extracting and interpreting data.
Add several numbers mentally.

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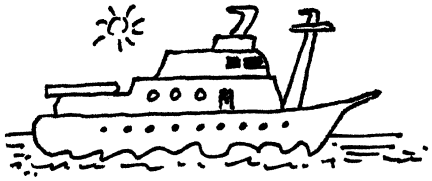
Slick Jim

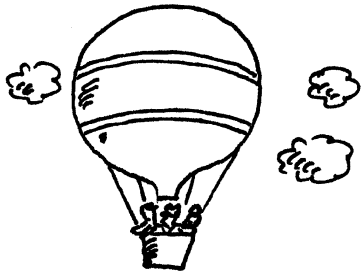
Slick Jim won the lottery.

He spent two thirds of his winnings on a very posh house.

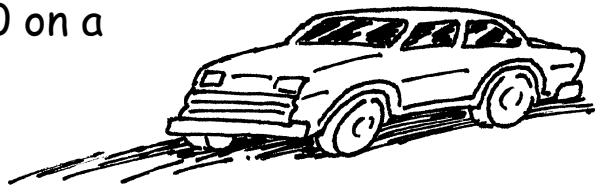
He spent two thirds of what he had left on a luxury yacht.



Then he spent two thirds of what he had left on a hot air balloon.



He spent his last £20000 on a flashy car.



How much did Slick Jim win on the lottery?

Teaching objectives

Solve a problem by organising information.
Find fractions of quantities.
Understand the relationship between multiplication and division.

76

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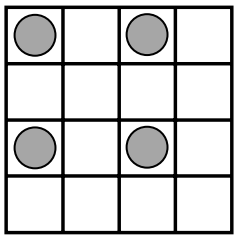
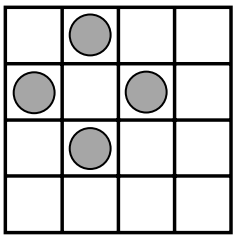
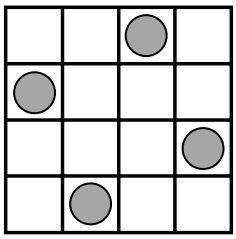
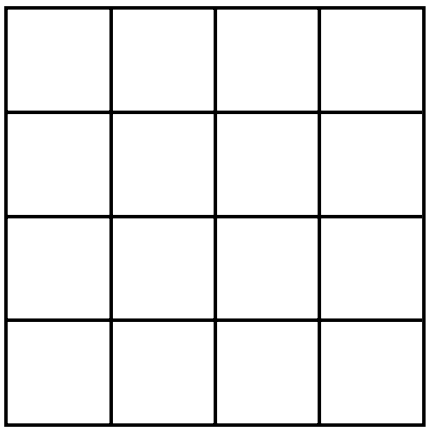
All square

On each of these grids, the counters lie at the four corners of a square.

What is the greatest number of counters you can place on this grid without four of them lying at the corners of a square?



77

Teaching objectives

Solve a problem by organising information.
Visualise 2-D shapes.

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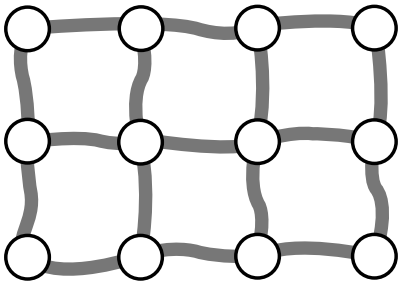
Sleigh ride

In Snow Town, 3 rows of 4 igloos are linked by 17 sleigh paths.

Each path is 10 metres long.



When Santa visits, he likes to go along each path at least once.

His route can start and end at any igloo.

How long is the shortest route Santa can take?

What if there are 4 rows of 5 igloos?

Teaching objectives

Solve a problem by organising information.
Visualise 2-D shapes.

78

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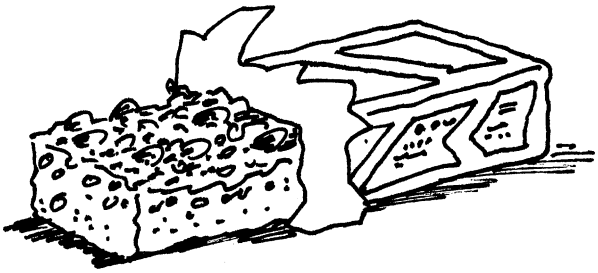
Spendthrift

Choc bars cost 26p each.

Fruit bars cost 18p each.



Anil spent exactly £5 on a mixture of choc bars and fruit bars.

How many of each did he buy?

79

Teaching objectives

Solve mathematical problems or puzzles.
Choose and use efficient calculation strategies to solve a problem.
Add sums of money.

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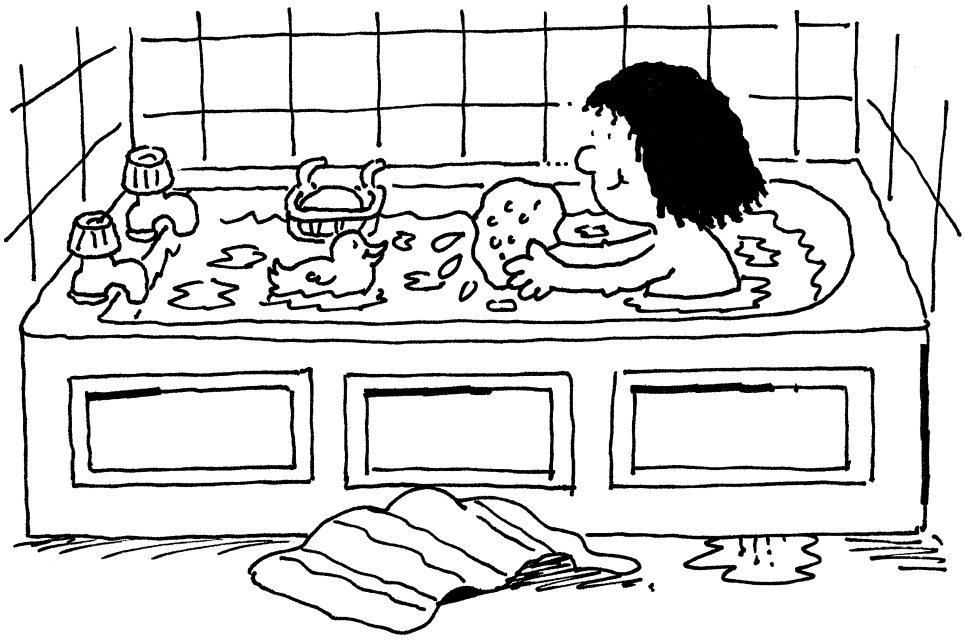
Cola in the bath

A can of cola holds 33 centilitres.

If you had a bath in cola – don't try it! – approximately how many cans of cola would you need?

Hint: 1 cubic centimetre is the same as 1 millilitre.



Teaching objectives

Solve mathematical problems or puzzles.
Estimate lengths and convert units of capacity.
Develop calculator skills and use a calculator effectively.

80

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Millennium

At what time of what day of what year will it be:

a. 2000 seconds
b. 2000 minutes
c. 2000 hours
d. 2000 days
e. 2000 weeks

after the start of the year 2000?

81

Teaching objectives

Solve mathematical problems or puzzles.
Convert smaller to larger units of time.
Develop calculator skills and use a calculator effectively.

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People in the crowd

Estimate how many people there are in the crowd.

Teaching objectives

Solve mathematical problems or puzzles.
Count larger collections by grouping.
Give a sensible estimate.

82

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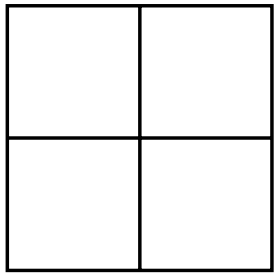
Make 200

1 2 3 4 5 6 7 8 9

Choose four of these digits.
Each one must be different.
Put one digit in each box.



This makes two 2-digit numbers reading across and two 2-digit numbers reading down.
Add up all four of the numbers.

In this example the total is 100.

1	2
4	7

12 + 47 + 14 + 27 = 100

How many different ways of making 200 can you find?

83

Teaching objectives

Solve mathematical problems or puzzles.
Know what each digit represents.
Add several two-digit numbers.

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Solutions

1 Four-pin bowling

Score 5 by knocking down 1 and 4, or 2 and 3.

Score 6 by knocking down 2 and 4, or 1, 2 and 3.

Score 7 by knocking down 3 and 4, or 1, 2 and 4.

2 Gob-stopper

Five different ways to pay 6p:

5p + 1p
2p + 2p + 2p
2p + 2p + 1p + 1p
2p + 1p + 1p + 1p + 1p
1p + 1p + 1p + 1p + 1p + 1p

Six different ways to pay 7p:

5p + 2p
5p + 1p + 1p
2p + 2p + 2p + 1p
2p + 2p + 1p + 1p + 1p
2p + 1p + 1p + 1p + 1p + 1p
1p + 1p + 1p + 1p + 1p + 1p + 1p

3 Pick a pair

There are six different sums and six different (positive) differences.

1. 1 + 2 = 3
   1 + 4 = 5
   2 + 4 = 6
   1 + 8 = 9
   2 + 8 = 10
   4 + 8 = 12

2. 2 – 1 = 1
   4 – 2 = 2
   4 – 1 = 3
   8 – 4 = 4
   8 – 2 = 6
   8 – 1 = 7

Adapt the puzzle by using larger numbers.

4 Snakes and ladders

Watching out for snakes, there are four different ways to get to 16 in two throws:

1 then 6; 3 then 4; 4 then 3; 5 then 2.

5 Bean-bag buckets

1. The highest score is 12 (3 bags in 4).
2. Score 6 in three ways:
   1 bag in 4 and 2 bags in 1, or
   1 bag in 1, 1 bag in 2 and 1 bag in 3, or
   3 bags in 2.
3. Score 9 in three ways:
   1 bag in 1 and 2 bags in 4, or
   1 bag in 2, 1 bag in 3, 1 bag in 4, or
   3 bags in 3.
4. Besides 6, 9 and 12, other possible scores are:

   3: 3 bags in 1

   4: 2 bags in 1, 1 bag in 2

   5: 2 bags in 1, 1 bag in 3, or
   1 bag in 1, 2 bags in 2

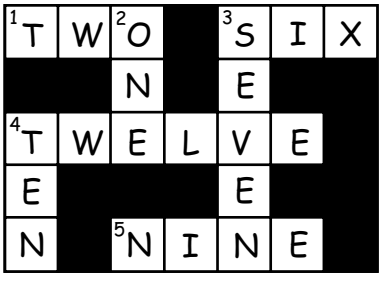
   7: 1 bag in 1, 2 bags in 3, or
   2 bags in 2, 1 bag in 3, or
   1 bag in 1, 1 bag in 2, 1 bag in 4

   8: 2 bags in 2, 1 bag in 4, or
   1 bag in 2, 2 bags in 3, or
   1 bag in 1, 1 bag in 3, 1 bag in 4

   10: 1 bag in 2, 2 bags in 4

Adapt this puzzle by using larger numbers.

6 Crossword



7 Gold bars

Move two bars from pile 1 to pile 3.
Move one bar from pile 4 to pile 2.

8 Ride at the fair

The amounts up to 20p that **cannot** be made from exactly three coins are:

1p, 2p, 10p, 18p, 19p.

Lucy could have given her Mum:

3p = 1p + 1p + 1p
4p = 2p + 1p + 1p
5p = 2p + 2p + 1p
6p = 2p + 2p + 2p
7p = 5p + 1p + 1p
8p = 5p + 2p + 1p
9p = 5p + 2p + 2p
11p = 5p + 5p + 1p
12p = 5p + 5p + 2p
13p = 10p + 2p + 1p
14p = 10p + 2p + 2p
15p = 5p + 5p + 5p
16p = 10p + 5p + 1p
17p = 10p + 5p + 2p

9 Sum up

If each number can be used only once:

9 = 2 + 3 + 4
10 = 2 + 8
11 = 3 + 8
12 = 4 + 8
13 = 2 + 3 + 8
14 = 2 + 4 + 8
15 = 3 + 4 + 8

Other solutions are possible if numbers can be repeated.

Other totals:

5 = 2 + 3
6 = 2 + 4
7 = 3 + 4
17 = 2 + 3 + 4 + 8

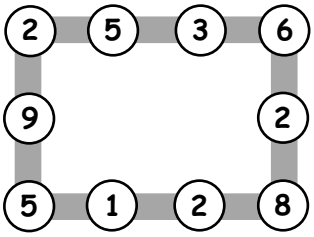
10 Birds' eggs

There are 10 possibilities:

1, 1, 17	1, 7, 11	3, 3, 13	5, 5, 9
1, 3, 15	1, 9, 9	3, 5, 11	5, 7, 7
1, 5, 13		3, 7, 9	

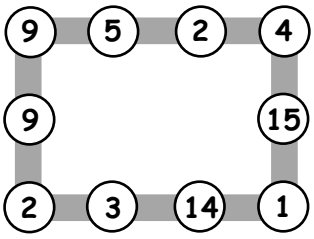
11 Number lines

1. For example:



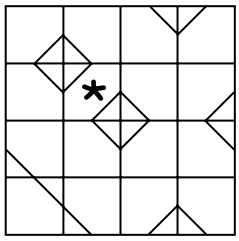
Other solutions are possible.

2. For example:

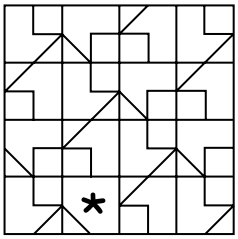


12 Odd one out

1.



2.



13 Line of symmetry

There are five other ways for Gopal to arrange the squares:

red, green, blue, blue, green, red
green, red, blue, blue, red, green
green, blue, red, red, blue, green
blue, red, green, green, red, blue
blue, green, red, red, green, blue

What if Gopal has eight squares: two red, two blue, two green and two yellow? How many different symmetrical lines can he make now? (24)

14 Card sharp

1. There are 10 different ways to choose three cards with a total of 12:

0, 3, 9	1, 2, 9	2, 3, 7	3, 4, 5
0, 4, 8	1, 3, 8	2, 4, 6	
0, 5, 7	1, 4, 7		
	1, 5, 6		

2. There are 9 different ways to choose four cards with a total of 12:

0, 1, 2, 9	0, 2, 3, 7	1, 2, 3, 6
0, 1, 3, 8	0, 2, 4, 6	1, 2, 4, 5
0, 1, 4, 7	0, 3, 4, 5	
0, 1, 5, 6		

3. No.

Adapt the puzzle by changing the total.

15 Jack and the beanstalk

Jack can climb the beanstalk like this:

left, left, right, right
left, right, left, right (as shown)
left, right, right, left
right, left, right, left
right, left, left, right
right, right, left, left

16 Monster

Alesha can use these coins to pay 45p:

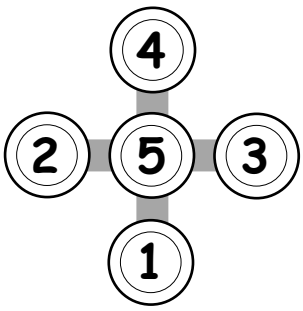
two 20p and one 5p
one 20p, two 10p and one 5p
one 20p, one 10p and three 5p
one 20p and five 5p
four 10p and one 5p
three 10p and three 5p
two 10p and five 5p
one 10p and seven 5p
nine 5p

There are 13 different ways to pay 50p using only silver coins. First add 5p to each of the ways for 45p. The other four possibilities are:

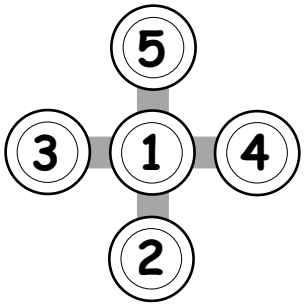
two 20p and one 10p
one 20p and two 10p
five 10p
one 50p

17 Cross-road

Each line adds up to 10.



Each line adds up to 8.



18 Fireworks

For 19 stars:

5 fireworks made 3 stars and 1 made 4 stars, or

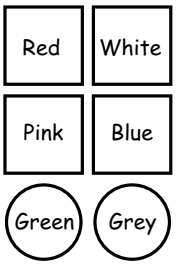
1 firework made 3 stars and 4 made 4 stars

For 25 stars:

3 fireworks made 3 stars and 4 fireworks made 4 stars, or

7 fireworks made 3 stars and 1 firework made 4 stars

19 Coloured shapes



20 Ones and twos

Some higher scores:

2 x 2 x 2 = 8	2 + 1 = 3
1 + 1 + 1 = 3	2 + 1 = 3
8 x 3 = 24	2 + 1 = 3
	3 x 3 x 3 = 27

21 Birthdays

Answer: Paul is 15.

Most pupils will guess then try to improve. For example, try 10:

10 x 2 = 20 20 + 5 = 25 too small

22 Christmas tree

There are 16 different ways:

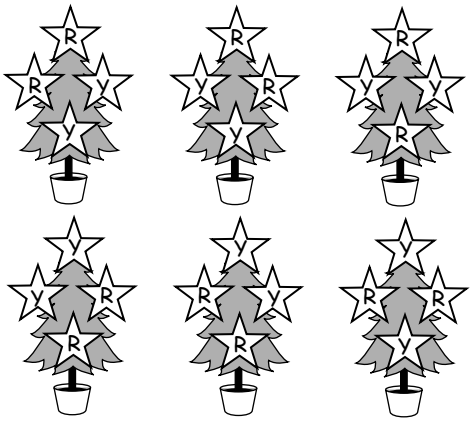
1 way for 4 red;

1 way for 4 yellow;

4 ways for 3 red and 1 yellow;

4 ways for 1 red and 3 yellow;

6 ways for 2 red and 2 yellow (shown below).



23 At the toyshop

There are 9 tricycles and 2 go-carts, or 4 tricycles and 5 go-carts.

24 Ben's numbers

There are 16 different numbers in Ben's list:

5, 14, 23, 32, 41, 113, 122, 131, 212, 221, 311, 1112, 1121, 1211, 2111, 11111.

What if the digits add up to 4, or if they add up to 6? How many different numbers are there now?

25 Spot the shapes 1

1. There are 9 triangles.
2. There are 18 rectangles.

26 Rows of coins

1. 5p, 2p, 20p, 1p, 10p
2. 2p, 5p, 1p, 2p, 1p, 5p, or its reverse
   When two 10p coins are also used:
   2p, 5p, 10p, 2p, 1p, 5p, 1p, 10p, or its reverse

27 Roly poly

1. The total number of dots on the dice is 21. Of these dots 17 are showing, so the face with 4 dots is face down.
2. The total number of dots on two dice is 42, so 12 dots are hidden. The two hidden faces must each have 6 dots.

28 Dan the detective

1. 48
2. 63

29 Spaceship

3 Tripods (9 legs) and 7 Bipods (14 legs), or 5 Tripods (15 legs) and 4 Bipods (8 legs).

What if Tripods with 3 legs and Quadrapods with 4 legs are on the spaceship?

Find two different ways to make 23 legs.

30 Susie the snake

Susie has 19 eggs.

You could make up similar problems with, say, 21 eggs.

If you counted them in fours, there would be 1 left over.

If you counted them in fives, there would be 1 left over.

31 Three monkeys

There are 10 possibilities:

1, 3, 21	3, 5, 17
1, 5, 19	3, 7, 15
1, 7, 17	3, 9, 13
1, 9, 15	5, 7, 13
1, 11, 13	5, 9, 11

What if the monkeys ate 24 nuts, with each of them eating a different even number of nuts?

The possible answers are:

2, 4, 18	4, 6, 14
2, 6, 16	4, 8, 12
2, 8, 14	6, 8, 10
2, 10, 12	

32 Card tricks

Systematic working helps to make sure that all possibilities have been considered.

Four different cards with a total of 20 are:

1, 4, 7, 8	2, 3, 7, 8	3, 4, 5, 8
1, 5, 6, 8	2, 4, 6, 8	3, 4, 6, 7
	2, 5, 6, 7	

Three different cards with a total of 16 are:

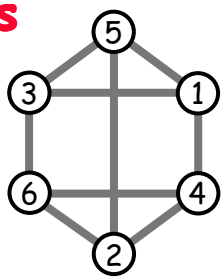
1, 7, 8	2, 6, 8	3, 5, 8	4, 5, 7
		3, 6, 7	

You could try other totals. For example, four cards with a total of 18 are:

1, 2, 7, 8	2, 3, 6, 7	3, 4, 5, 6
1, 3, 6, 8	2, 4, 5, 7	
1, 4, 5, 8		
1, 4, 6, 7		

Explore the different totals that can be made with four cards. (It is possible to make any total from 10 to 26.)

33 Neighbours



Here is one possible solution.

Can you find others?

34 Queen Esmeralda's coins

There were 7, 3, 4 and 6 coins in each pile.

The problem can be solved by trial and error.

35 Duck ponds

1. 2 2 5 5

2. 2 4 8

3. 5 4 3 2

You could try similar problems with other numbers. For example, using 15 ducks and

5 ponds	make each hold 1 more than the one before (1, 2, 3, 4, 5)
4 ponds	make each hold twice as many as the one before (1, 2, 4, 8)
3 ponds	make each hold 4 more than the one before (1, 5, 9)
3 ponds	make each hold 2 less than the one before (7, 5, 3)

36 Treasure hunt

Jed	R	D	E	R D	R E	E D	R E D	
Jake	E D	E R	D R	E	D	R		E D R

37 Stamps

Tilly stuck three 10p stamps and five 5p stamps on her parcel.

No. of 5p stamps	No. of 10p stamps	Total value
8	0	40p
7	1	45p
6	2	50p
5	3	55p
4	4	60p
3	5	65p
2	6	70p
1	7	75p
0	8	80p

To adapt the problem, change the cost of the parcel, or use different stamps.

38 Maisie the mouse

Maisie had 46 breadcrumbs.

The problem can be solved by experiment.

Alternatively, list all the multiples of 4. Add 2 to each number in the list.

Now list all the multiples of 5. Add 1 to each number in the list.

Now look for a number lying between 30 and 50 that is common to both lists.

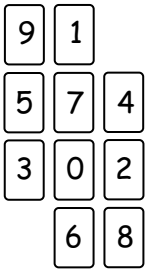
To adapt the problem, group the breadcrumbs in 5s and 6s, or 7s and 9s.

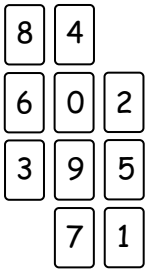
39 Kieron's cats

Kieron's cats weigh 5 kg, 2 kg and 6 kg.

40 Next door numbers

For example:





41 Nick-names

Dawn is Ace.
Mark is Curly.
Josh is Fudgy.
Tina is Spider.

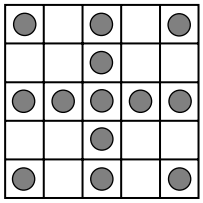
42 Stickers

There are 8 stickers in a full sheet.

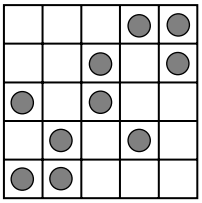
43 Odds and evens

Several solutions are possible. For example:

1.



2.



44 More stamps

Rosie bought four 20p stamps and twelve 10p stamps.

45 Sandcastles

Over the 5 days Lisa made 24, 20, 16, 12 and 8 sandcastles.

She made 84 sandcastles altogether.

46 Sail away

Two women cross the river together.
One woman stays there and one brings the boat back.

One man crosses the river.
One woman brings the boat back.

Two women cross the river together.
One woman stays there and one brings the boat back.

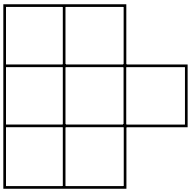
The second man crosses the river.
One woman brings the boat back.

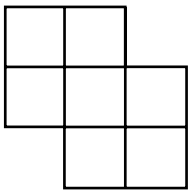
Two women cross the river together.

47 Straw squares

You can make a maximum of 9 squares with 20 straws.

Here are two ways of doing it.





For older children, try 40 straws.

With these you can make a maximum of 30 squares.

48 King Arnold

Three knights can sit with King Arnold in 6 different ways.

Four knights can sit with King Arnold in 24 different ways.

49 Footsteps in the snow

Counting from zero in 2s, 3s and 5s will first match up at 30, when Little has taken 15 footsteps.

50 Ski lift

The ski lift has 180 chairs.

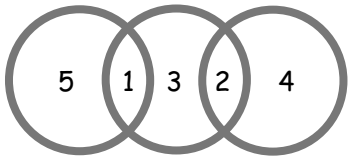
51 Lighthouses

All three lights will be off after 5 seconds.

All three lights will next come on together after 120 seconds.

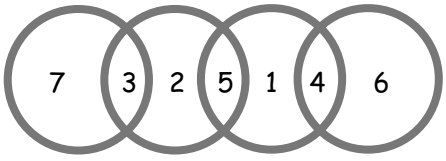
52 Circle sums

1.



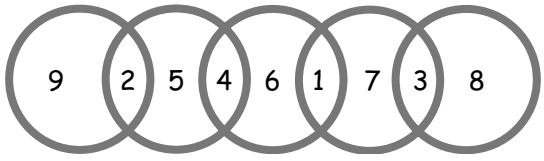
or its reverse

2.



or its reverse

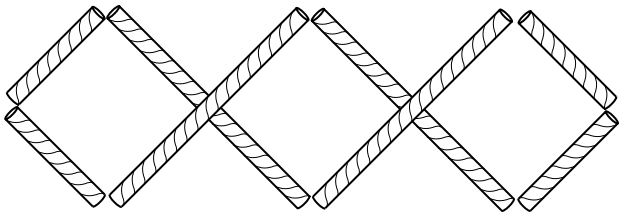
3.



or its reverse

53 Square it up

For example:



54 Joins

Using four numbers:
the highest score is 19 + 15 + 17 + 18 = 69,
the lowest score is 6 + 5 + 2 + 17 = 30.

Using five numbers:
the highest is 20 + 18 + 13 + 17 + 18 = 86,
the lowest is 6 + 18 + 2 + 5 + 6 = 37.

Using five numbers and diagonal joins:
the highest is 19 + 17 + 14 + 15 + 18 = 83,
the lowest is 13 + 6 + 20 + 2 + 6 = 47.

55 Money bags

Ram put 1p, 2p, 4p and 8p in the four bags.

Any sum from 1p to 15p can be made with these amounts.

56 A perfect match

1. A matchbox tray fits into its outer cover in 4 different ways.

2. A cube will fit into a box with any one of its 6 faces uppermost.

   Each face can be rotated into any one of 4 different positions.

   So there are 6 x 4 = 24 ways of fitting the cube in the box.

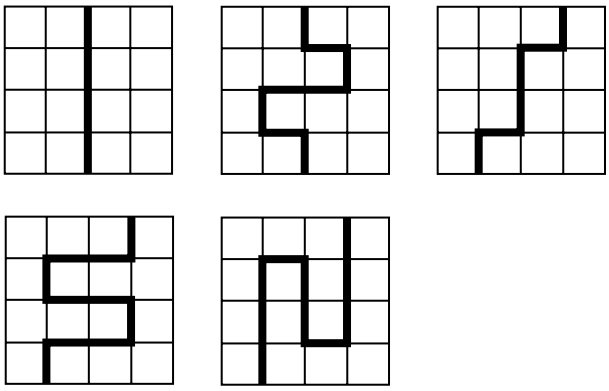
57 Presents

Gurmit paid £2, £4, £6, £1 and £8 for the five presents.

58 Spot the shapes 2

1. There are 11 triangles.
2. There are 17 squares.

59 Four by four



60 Three digits

You can make six different numbers.
In order, the numbers are:
799, 889, 898, 979, 988, 997.

61 Make five numbers

For example:
a. 12, 39, 45, 60, 78.
b. 7, 42, 63, 98, 105.
c. 5, 23, 67, 89, 401.
There are other solutions.

62 Maze

There are two routes that total 100 exactly:

+ 6 x 7 − 6 x 3 − 8 = 100
+ 9 x 7 ÷ 3 x 5 − 5 = 100

The route giving the highest total is:

+ 9 x 7 − 6 x 7 − 8 = 391

The route giving the lowest total is:

+ 6 x 7 ÷ 3 x 3 − 8 = 34

63 Jack's book

The book has 221 pages.
42 of the digits are a 5.

64 Flash Harry

Flash Harry's bank balance looked like this.

April	− £100
May	+ £100
June	− £200
July	+ £200

So Harry made £200 overall.

65 Age old problems

1. I am 48 years old (or possibly 104).
2. I am now 26 years old. In 38 years' time, when I am 64, my age will be both a square number and a cube.
3. I am 9 years old now.

66 Zids and Zods

There are 3 Zids with 4 spots and 4 Zods with 9 spots.

If Zids have 5 spots and Zods have 7 spots, the possible ways of making 140 are:

28 Zids;
21 Zids and 5 Zods;
14 Zids and 10 Zods;
7 Zids and 15 Zods;
20 Zods.

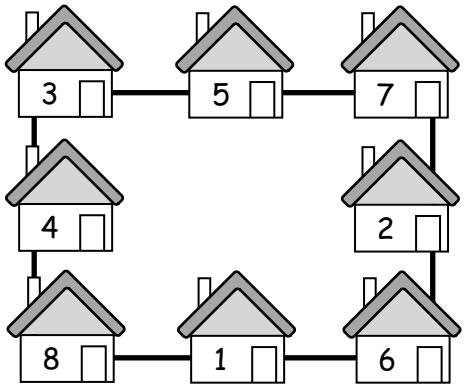
67 Franco's fast food

A curry costs £3.50, a pudding costs £1 and a tea costs 50p.

So the total cost of a curry, a pudding and a tea is £5.

68 Albert Square

For example:



69 Coins on the table

Anna put 12 coins on the table.

70 A bit fishy

Nasreen bought 4 angel fish and 8 goldfish.

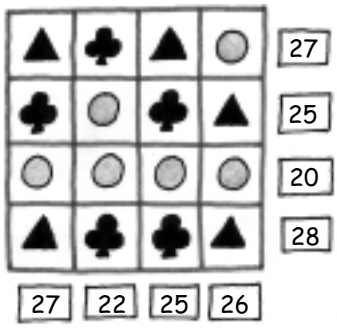
71 Pet shop

1. Jim sold the dog and the cat for £72 and £48 respectively, a total of £120.
2. The dog cost £50 and the cat cost £75, a total of £125.

   The cat and the dog were sold for a total of £120, so Jim made a loss of £5.

72 Shape puzzle

The circle has the value 5.
The triangle has the value 8.
The club has the value 6.



73 Eggs

Mrs Choy bought:
10 large eggs at 50p each,
10 medium eggs at 10p each,
80 small eggs at 5p each.

74 Anyone for tennis?

Ali, Luke, Holly and Zoe play tennis.

Two boys can play.
Ben won't play if Luke plays.
So the two boys must be Ali and Ben, or Ali and Luke.

Ali will play only if Holly plays.
Holly won't play with Ben.
So the two boys are Ali and Luke.

Luke will play only if Zoe plays.
So the two girls are Holly and Zoe.

75 Bus routes

There are six different routes from A back to A:

A B C D E F A
A B D C E F A
A B D E C F A

and the three reversals of these.

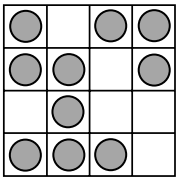
The cheapest routes are A B D E C F A and its reversal, which each cost £21.

76 Slick Jim

Jim won £540 000.

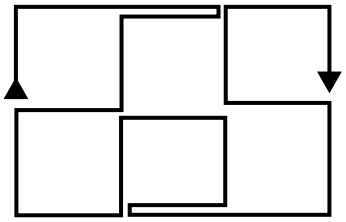
77 All square

For example:

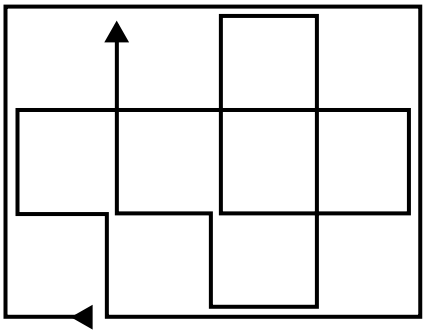


78 Sleigh ride

With 3 rows of 4 igloos, the shortest route is 190 metres. For example:



With 4 rows of 5 igloos, the shortest route is 350 metres. For example:



79 Spendthrift

Anil bought 13 choc bars and 9 fruit bars, or 4 choc bars and 22 fruit bars.

80 Cola in the bath

A bath 1.5 metres long by 60 cm wide would have a floor area of approximately 9000 cm^2. If there was 10 cm of cola in the bath, the volume of liquid would be about 90 000 cm^3 or 90 000 ml. This would require roughly 270 cans of cola.

81 Millennium

a.	00:33:20	1 January	2000
b.	09:20:00	2 January	2000
c.	08:00	23 March	2000
d.	00:00	23 June	2005
e.	00:00	1 May	2038

82 People in the crowd

There is no precise answer, but pupils can compare their estimates and discuss how they arrived at them.

83 Make 200

There are 22 different solutions. Eleven of the solutions are as follows:

1	9	2	8	2	9	3	5
7	2	6	3	5	3	7	4

4	1	4	2	4	3	5	1
9	5	8	5	7	5	7	6

6	1	6	2	7	1
5	7	4	7	3	8

Eleven more solutions are formed by changing over the two digits in the top right and bottom left boxes.