



INVESTIGATION



10 × 10 Multiplication Square

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2	4	6	8	10	12	14	16	18	20
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5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
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9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

MathSphere

10 × 10 Multiplication Square

Here is a 10 × 10 multiplication square:

×	1	2	3	4	5	6	7	8	9	10
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2	2	4	6	8	10	12	14	16	18	20
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4	4	8	12	16	20	24	28	32	36	40
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This investigation uses only the 'answers' in the table, so we can remove the left column and the top row. This gives us:

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The Problem

Your task is to find and explain as many patterns as you can within this multiplication square.

Are there many patterns?

Many?! There are thousands!!!!

Good Advice:

Work in a methodical way, recording your results carefully as you go. Try to explain the patterns you find. This is not always easy, but explain the ones you can.

It helps to have a number of the table squares already photocopied for your use. Cut out the sections you need. You do not need to use a whole square if you are just looking at one small section.

Some ideas:

1. Look at the rows and columns and see if you can see any patterns in these. For example, what do you notice about the sixth column?
2. Look at 2×2 squares taken from the middle of the table square and see what you can spot.

Eg.

6	8
9	12

You could try multiplying diagonally across this small square:

6×12 and 8×9 .

Did you notice anything?

Does this work for any 2×2 square?

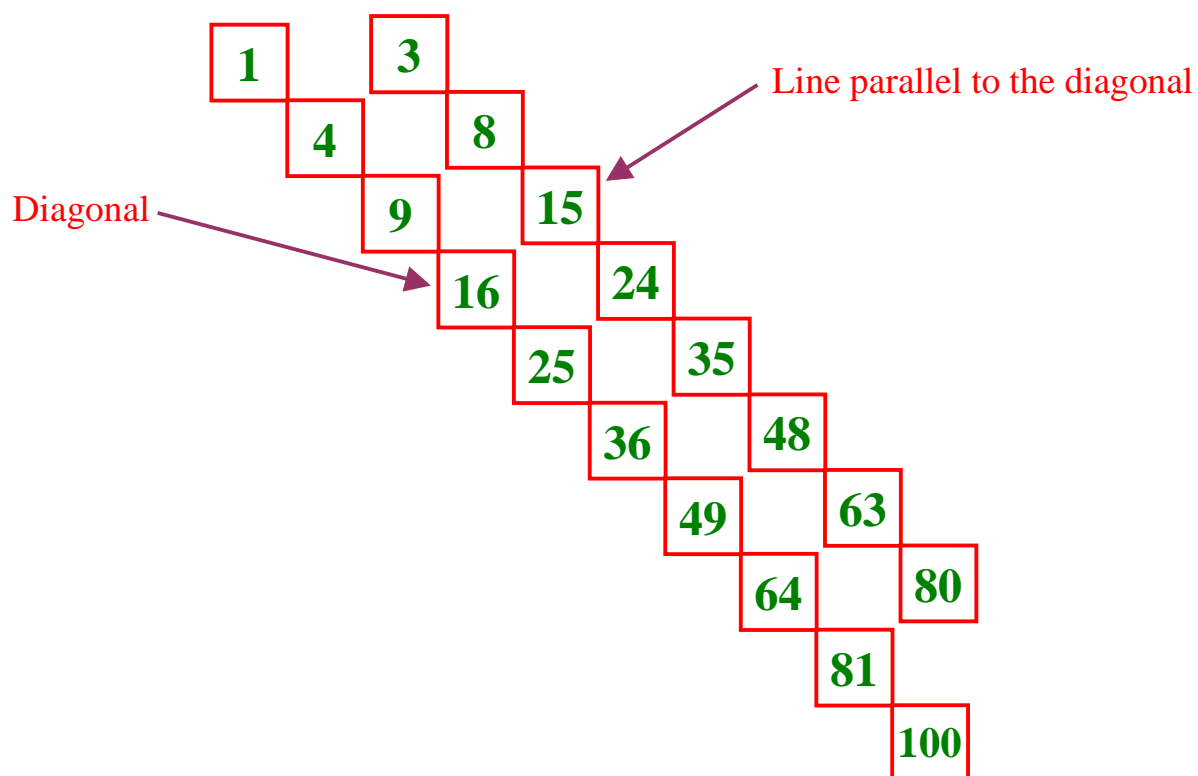
Some ideas continued:

3. You could try this with 3×3 squares:

5	6	7
10	12	14
15	18	21

Is there anything else you could try with a 3×3 square?

4. Do the things you have discovered work in small squares of all sizes, 4×4 , 5×5 etc ?
5. You could look at the diagonals of the square and lines on numbers parallel to the diagonals.



6. When you have found a good sequence of numbers, you could try to draw a graph of them. For instance, the numbers on the diagonal above make a good graph. Do you recognise these numbers?

Some ideas continued:

7. You could try working in parts of two rows or columns, like this:

4	8	12	16
5	10	15	20

8. You could look for all the multiples of different numbers and see if you can discover any patterns. For example, what happens if you colour in all the multiples of 6 ?
9. Try colouring all the squares that are less than a certain number.
Eg. Colour all the numbers less than 30.
Where are these numbers on the square?
10. What happens if you multiply all the numbers in the third column by the numbers in the fourth column? What do you have to do to your table square to help you with this?

Thanks! I think I have got enough ideas to get me started now.

Enjoy your work and record your results properly.

Try to find as many rules and patterns as you can.

Answer Guide

Here are some possible answers and notes for guidance.

This is a great investigation for children of all abilities and ages as there is so much to discover and to try to prove.

Do encourage children to investigate their own ideas, but they must work methodically and record proficiently if they are to make sense of their efforts.

1. Obviously the rows and columns are the respective times tables, but it is surprising how many children fail to understand why this is, even if the rows and columns are key elements in a multiplication table!
2. 2×2 squares always cross multiply to give the same answer on both diagonals.

Suppose we take the square in the illustration. The 6 comes from 2×3 , the 8 from 2×4 , the 9 from 3×3 and the 12 from 3×4 .

This means that 6×12 is really $2 \times 3 \times 3 \times 4$
and 8×9 is really $2 \times 4 \times 3 \times 3$.

It is not difficult to see that each multiplication contains the same product and must therefore give the same answer.

If the diagonal numbers are added instead of multiplied, one diagonal is always one less than the other. Always true, but needs a bit of algebra to prove.

3. This is an extension to larger squares. It is not difficult to find similar relationships in 3×3 squares.
4. A natural development from 1. and 2.
5. The numbers on the leading diagonal are the square numbers.

Some children may be able to work out the general term for the sequence. This is, of course, n^2 , where n is the number of the term in the sequence.

Other diagonal lines can provide interesting sequences.

6. the square numbers make a very interesting graph. Plot the numbers from 1 to 10 evenly on the horizontal graph and then the numbers 1 to 100 on the vertical axis. Try graphs of the table numbers, 3, 6, 9, 12, 15 etc for the times table.

Answer Guide (Contd)

7. For rectangles children can try multiplying the corner numbers, but can also look at relationships between the rows. Is it possible to predict the sum of the numbers in the rectangle if you know the shape of the rectangle and the top left corner number?

There are so many ideas children can try with this problem, it is often best to let them develop their own ideas within the bounds you wish to set them.

On the following page there is a set of table squares which you may copy for the children to cut up and stick in their work. This saves hours of copying out the table by hand. Some children would happily do that for two weeks and not produce anything with any mathematical content!

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