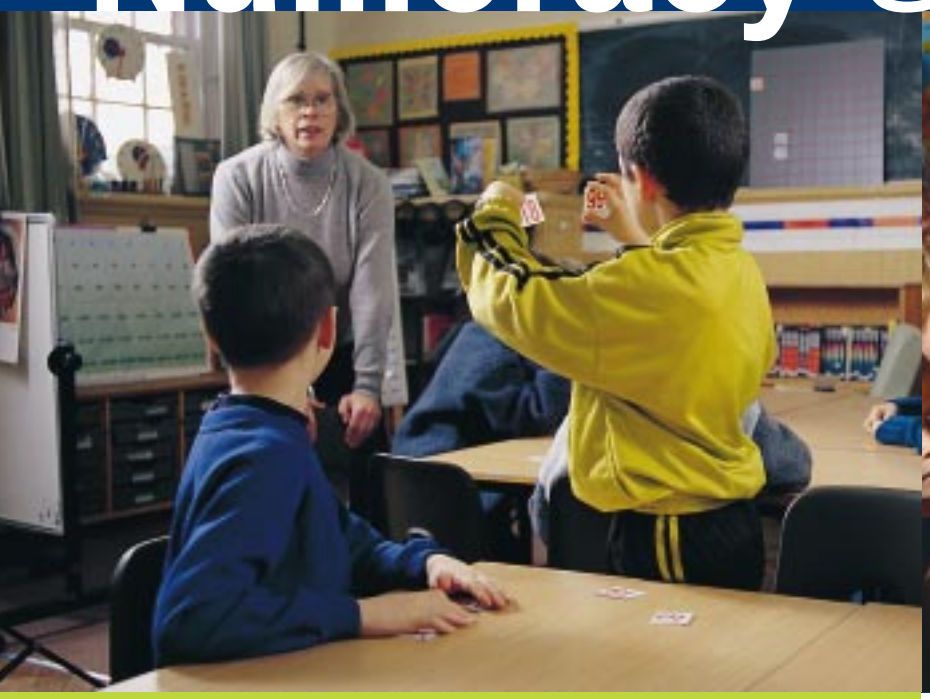
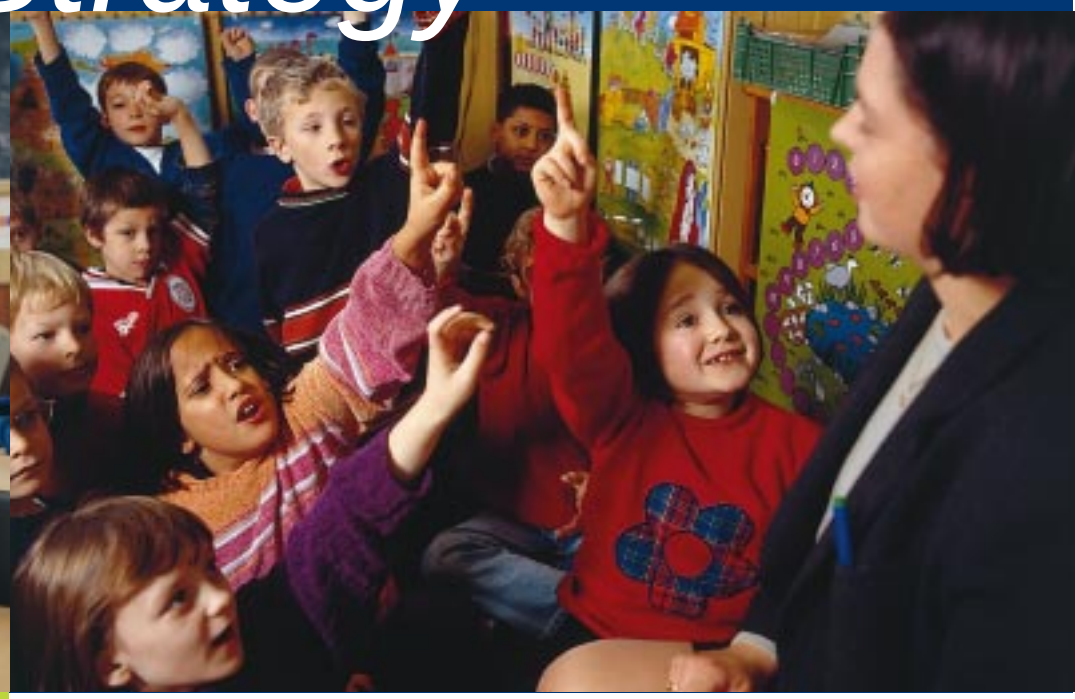
Raising Standards
Standards and Effectiveness Unit

The National **Numeracy** *Strategy*





Framework for teaching mathematics
from Reception to Year 6

DfEE
Department for
Education and Employment

Introducing the Framework

Foreword

Numeracy is a key life skill. Without basic numeracy skills, our children will be disadvantaged throughout life. That is why we have set a target of 75% of 11-year-olds reaching the standard of mathematics expected for their age by 2002. And, through the National Numeracy Strategy, we have made funding available to provide training and support for schools as they work towards this target. The National Numeracy Strategy will be launched in schools this September and will build on the experience of the National Literacy Strategy. Through these two mutually reinforcing initiatives, we want children to achieve a secure foundation in literacy and numeracy as we move into the next millennium.

This *Framework for Teaching Mathematics from Reception to Year 6* is a key part of the package of support available for teachers, headteachers and governors. Schools which have already tested it out have found that the yearly teaching programmes, key objectives and planning grids greatly assist them in planning and teaching their mathematics lessons. I believe that the Framework has much to offer every school.

We recognise the crucial importance of giving teachers the support they need to raise standards. But the success of both the Literacy and Numeracy Strategies depends on the commitment and enthusiasm of teachers themselves. I hope that all schools will use the Framework to make a real difference.

The Rt Hon David Blunkett MP
Secretary of State for Education and Employment
March 1999

Introduction

Over the past few years an accumulation of inspection, research and test evidence has pointed to a need to improve standards of literacy and numeracy. The Government's targets for 2002 are that 80% of all 11-year-olds achieve at least level 4 in the National Curriculum tests for English, and 75% achieve at least level 4 in the tests for mathematics. The National Literacy Strategy has been launched, with a *Framework for Teaching Literacy* in a daily literacy hour, and a programme of in-service training and support for schools.

The National Numeracy Strategy will complement the Literacy Strategy. From September 1999, schools will provide a structured daily mathematics lesson of 45 minutes to one hour for all pupils of primary age. Teachers will teach the whole class together for a high proportion of the time, and oral and mental work will feature strongly in each lesson.

This *Framework for Teaching Mathematics from Reception to Year 6* is a key element of the training materials that will support the Numeracy Strategy.

The Framework illustrates the intended range and balance of work in primary mathematics to make sure that pupils become properly numerate. It is intended mainly for day-to-day reference by class teachers and has been tested extensively and successfully in schools of different types and sizes. Its purpose is to help primary and middle schools, and special schools with primary-age pupils, to set appropriately high expectations for their pupils and understand how pupils should progress through the primary years. Teachers working with older pupils who have more severe or complex special educational needs might also find its contents relevant.

Good school leadership is vital for teachers to be successful in the classroom. The Framework should also be read and used by headteachers and governors as they manage the improvement of standards of mathematics, monitor progress and support the staff. Parents too may be interested to see it.

The contents of the Framework

The National Curriculum Order describes what must be taught in each key stage. This Framework provides guidance to supplement the Order. It has been developed alongside the proposals for the revised National Curriculum so that it is compatible with them.

The Framework contains a set of **yearly teaching programmes** illustrating how mathematics can be planned and taught from Reception to Year 6. It includes guidance on the daily mathematics lesson in which this teaching will take place and on the assessment of pupils' progress.

The objectives in the yearly teaching programmes cover all aspects of the National Curriculum for mathematics in Key Stages 1 and 2. The programme for Reception takes account of the Early Learning Goals for 3- to 5-year-olds, and provides a bridge from the Goals to the National Curriculum.

Some of the objectives in the yearly programmes are more critical than others if children are to become numerate. These **key objectives** are listed in a separate section and are also highlighted in bold type in the yearly teaching programmes. They are central to all pupils' achievements in relation to the National Curriculum level descriptions, and hence to their performance in teacher assessments and tests. Teachers should give priority to these key objectives when they are planning work and assessing pupils' progress.

Each yearly teaching programme is accompanied by grids to help teachers to plan a term's lessons in outline. The **planning grids** show how mathematical topics can be grouped in units of work throughout the term. A recommended number of lessons is given for each unit. Time is built in for half-termly assessment and review.

After the yearly teaching programmes and planning grids come **supplements of examples**, for Reception, Years 1–3 and Years 4–6. These examples are not intended to be covered as a 'scheme of work', or used as a 'textbook' or for worksheets. Their purpose is to illustrate, for each teaching objective, a selection of what pupils should know, understand and be able to do by the end of each school year. Over time, teachers should add to and replace the examples with those that they find most useful and cross-reference them to other resources, including information and communications technology (ICT), to form a working document.

The section on planning starting on page 38 discusses in detail the key objectives, yearly teaching programmes, planning grids and examples, and explains how to use them.

Related publications

For schools that are starting to prepare for the National Numeracy Strategy there are a number of other publications to accompany the Framework.

- *The Implementation of the National Numeracy Strategy* This final report of the Numeracy Task Force, published in July 1998 by the DfEE, gives a practical agenda for action to implement the National Numeracy Strategy. It can be obtained from DfEE Publications, tel: 0845 6022260.
- *Mathematical Vocabulary* This 32-page booklet lists the important vocabulary for each year group. An introductory section discusses questioning strategies. Further copies of the booklet can be obtained from DfEE Publications, tel: 0845 6022260.
- *Teaching Mental Calculation Strategies: Guidance for Teachers at Key Stages 1 and 2* This book has been produced by the Qualifications and Curriculum Authority (QCA) to support the National Numeracy Strategy. It describes approaches to the teaching of mental calculations and the role of calculators in Key Stages 1 and 2. Further copies can be obtained from QCA Publications, tel: 01787 884444; fax: 01787 312950.
- *Standards in Mathematics: Exemplification of Key Learning Objectives from Reception to Year 6* This book, also produced by the QCA, describes the key learning objectives for each age group from Year 1 to Year 6 and illustrates them with examples of children's work. Further copies can be obtained from QCA Publications, tel: 01787 884444; fax: 01787 312950.

What is numeracy?

Numeracy is a proficiency which involves confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams, charts and tables.

As a teacher you can help children to acquire this proficiency by giving a sharp focus to the relevant aspects of the programmes of study for mathematics. The outcome should be numerate pupils who are confident enough to tackle mathematical problems without going immediately to teachers or friends for help. Your pupils should:

- have a sense of the size of a number and where it fits into the number system;
- know by heart number facts such as number bonds, multiplication tables, doubles and halves;
- use what they know by heart to figure out answers mentally;
- calculate accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies;
- recognise when it is appropriate to use a calculator, and be able to do so effectively;
- make sense of number problems, including non-routine problems, and recognise the operations needed to solve them;
- explain their methods and reasoning using correct mathematical terms;
- judge whether their answers are reasonable and have strategies for checking them where necessary;
- suggest suitable units for measuring, and make sensible estimates of measurements; and
- explain and make predictions from the numbers in graphs, diagrams, charts and tables.

Factors that promote high standards of numeracy

As part of the National Literacy Strategy your school will have considered ways of promoting higher standards of literacy. The action that you have already taken will help you when you come to consider ways of promoting pupils' numeracy skills, since many of the same factors will apply.

Where **school management** is concerned, better numeracy standards occur when:

- the headteacher is well-informed, provides active leadership and sets high expectations for what can be achieved by staff and pupils;
- a co-ordinator for mathematics has the expertise, opportunity and support needed to influence practice;
- a desire to secure high standards through effective teaching and learning pervades the whole school;
- there are clear, realistic targets for raising standards, and a manageable plan for achieving them, with regular evaluation of the school's progress towards the targets – including effective arrangements, which take account of national standards, for assessing the progress of whole year groups and each class;

- there is systematic monitoring and self-review, under the headteacher's direction, of teachers' planning, teaching and assessment;
- there is a whole-school approach to the professional development of teachers and other staff involved in the teaching of mathematics, with emphasis on developing knowledge of the primary mathematics curriculum and appropriate teaching methods;
- classroom assistants take part in planning and are used effectively to support teachers in mathematics lessons;
- parents are kept well-informed and encouraged to be involved through discussions at school and sometimes in work with pupils at home; and
- governors are involved actively in policy, monitoring and evaluation.

Taking the **curriculum and assessment** in the school as a whole, better numeracy standards occur when:

- staff share a common understanding of numeracy and how best to promote it;
- there is a daily, dedicated mathematics lesson in every class, with lesson time extended through out-of-class activities and regular homework;
- the teaching programme is based on identified learning objectives, and is planned thoroughly, to ensure high expectations, consistent approaches and good progression throughout the school;
- the foundations of mental calculation and recall of number facts are established thoroughly before standard written methods are introduced;
- assessments are used to identify pupils' strengths and difficulties, to set group and individual targets for them to achieve and to plan the next stage of work;
- assessments include informal observations and oral questioning, regular mental tests, and half-termly planned activities designed to judge progress; and
- recording systems give teachers the information that they need to plan and report successfully, but are not too time-consuming to maintain.

Where **teaching** is concerned, better numeracy standards occur when teachers:

- structure their mathematics lessons and maintain a good pace;
- provide daily oral and mental work to develop and secure pupils' calculation strategies and rapid recall skills;
- devote a high proportion of lesson time to direct teaching of whole classes and groups, making judicious use of textbooks, worksheets and ICT resources to support teaching, not to replace it;
- demonstrate, explain and illustrate mathematical ideas, making links between different topics in mathematics and between mathematics and other subjects;
- use and give pupils access to number lines and other resources, including ICT, to model mathematical ideas and methods;
- use and expect pupils to use correct mathematical vocabulary and notation;
- question pupils effectively, including as many of them as possible, giving them time to think before answering, targeting individuals to take account of their attainment and needs, asking them to demonstrate and explain their methods and reasoning, and exploring reasons for any wrong answers;
- involve pupils and maintain their interest through appropriately demanding work, including some non-routine problems that require them to think for themselves;
- ensure that differentiation is manageable and centred around work common to all the pupils in a class, with targeted, positive support to help those who have difficulties with mathematics to keep up with their peers.

The approach to calculation

An ability to calculate mentally lies at the heart of numeracy. You should emphasise mental methods from the early years onwards with regular opportunities for all pupils to develop the different skills involved. These skills include:

- remembering number facts and recalling them without hesitation;
- using the facts that are known by heart to figure out new facts: for example, a fact like 8 + 6 = 14 can be used to work out 80 + 60 = 140, or 28 + 6 = 34;
- understanding and using the relationships between the 'four rules' to work out answers and check results: for example, 24 ÷ 4 = 6, since 6 × 4 = 24;
- drawing on a repertoire of mental strategies to work out calculations like 81 – 26, 23 × 4 or 5% of £3000, with some thinking time;
- solving problems like: 'Can I buy three bags of crisps at 35p each with my £1 coin?' or: 'Roughly how long will it take me to go 50 miles at 30 m.p.h.?'

An emphasis on mental calculation does not mean that written methods are not taught in the primary years but the balance between mental and written methods, and the way in which pupils progress from one to the other, is very important.

The first stages

In the early years children will use oral methods, in general moving from counting objects or fingers one by one to more sophisticated mental counting strategies. Later they will use a number line or square to work out their answers in different ways, depending on the numbers involved. After giving them experience of a variety of situations, real and imagined, you should teach them to remember and recall simple number facts such as 5 add 3 is 8 or that 7 taken from 9 leaves 2. Posing problems and expressing relationships in different ways, and encouraging children to use this language when they talk about mathematics, is an important stage in developing their calculation strategies and problem-solving skills.

These early stages of mental calculation are not, however, at the exclusion of written recording. Alongside their oral and mental work children will learn first to read, interpret and complete statements like 5 + 8 = □ or 13 = □ + 5, and then to record the results of their own mental calculations in the correct way, using a horizontal format like 43 – 8 = 35. They should also be taught addition and subtraction alongside each other so that they are able to write the subtraction corresponding to a given addition sum, and vice versa.

The first stage of recording calculations

Pupils learn to read number statements and interpret signs and symbols. They write answers only, to develop or practise rapid recall. For example:

6 + 4 =	□ + 10 = 17
26 + 4 =	17 – □ = 10
36 + 4 =	

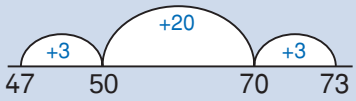
Larger numbers and informal jottings

As pupils progress to working with larger numbers they will learn more sophisticated mental methods and tackle more complex problems. They will develop some of these methods intuitively and some you will teach explicitly.

Through a process of regular explanation and discussion of their own and other people's methods they will begin to acquire a repertoire of mental calculation strategies. At this stage, it can be hard for them to hold all the intermediate steps of a calculation in their heads and so informal pencil and paper notes, recording some or all of their solution, become part of a mental strategy. These personal jottings may not be easy for someone else to follow but they are an important staging post to getting the right answer and acquiring fluency in mental calculation.

The next steps in recording calculations

Pupils make jottings to assist their mental calculations: e.g. 47 + 26



Pupils record steps so that you and they can see what they have done: e.g.

36 + 27

36 + 20 → 56 56 + 7 → 63

or

30 + 20 → 50 and 6 + 7 → 13

50 + 13 → 63

Not everyone does a mental calculation like 81 – 26 in the same way (nor is it necessary for them to do so) but some methods are more efficient and reliable than others. By explaining, discussing and comparing different part written, part mental methods, you can guide pupils towards choosing and using the methods which are most efficient and which can be applied generally. At this point, the need for more formal recording of calculation methods emerges.

Standard written methods

Standard written methods are reliable and efficient procedures for calculating which, once mastered, can be used in many different contexts. But they are of no use to someone who applies them inaccurately and who cannot judge whether the answer is reasonable. For each operation, at least one standard written method should be taught in the later primary years but the progression towards these methods is crucial, since they are based on steps which are done mentally and which need to be secured first. For example, the calculation of 487 + 356, done by the method which has been taught traditionally, requires the mental calculations 7 + 6 = 13, 8 + 5 + 1 = 14 and 4 + 3 + 1 = 8, while a division calculation such as 987 ÷ 23 can involve mental experiment with multiples of 23 before the correct multiple is chosen.

Most countries, and in particular those which are most successful at teaching number, avoid the premature teaching of standard written methods in order not to jeopardise the development of mental calculation strategies. The bridge from recording part written, part mental methods to learning standard methods of written calculations begins only when children can add or subtract reliably any pair of two-digit numbers in their heads, usually when they are about 9 years old.

Using standard written methods

Pupils write to work out complex calculations that they cannot do mentally: e.g.

$$\begin{array}{r} 253 \\ 576 \\ +\ 843 \\ \hline 1672 \end{array} \qquad 7\overline{)1624}\ 232$$

When they have reached the stage of working out more complex calculations using pencil and paper you should still expect your pupils to practise and develop their mental calculation strategies. When faced with any calculation, no matter how large or how difficult the numbers may appear to be, the first question pupils should always ask themselves is: 'Can I do this in my head?' They then need to ask themselves: 'Do I know the approximate size of the answer?' so that they can be reasonably sure their calculation is right.

The role of calculators

The calculator is a powerful and efficient tool. It has a strong part to play in subjects such as geography, history or science, since it allows children of primary age to make use of real data – often numbers with several digits – that they have gathered in their research or experiments, perhaps to work out a percentage, or to compare totals or proportions.

In the primary years, the calculator's main role in mathematics lessons is not as a calculating tool, since children are still developing the mental calculation skills and written methods that they will need throughout their lives. But it does offer a unique way of learning about numbers and the number system, place value, properties of numbers, and fractions and decimals. For example, you could use an overhead projector calculator for whole-class demonstration purposes so that the class can predict what happens when they multiply by 10 or divide by 10, or individual pupils might use a calculator to find two consecutive numbers with a given product and then discuss their different approaches.

If children are to use the basic facilities of a calculator constructively and efficiently, you need to teach them the technical skills they will require: the order in which to use the keys; how to enter numbers such as sums of money, measurements or fractions; how to interpret the display; how to use the memory… Children need to learn when it is, and when it is not, appropriate to use a calculator, and their first-line strategy should involve mental calculations wherever possible. For example, you might show pupils that they can 'beat the calculator' if they can recall number facts rapidly. They should also have sufficient understanding of the calculation in front of them to be able to decide which method to use – mental, pencil and paper, or calculator. When they do use a calculator they should be able to draw on well-established skills of rounding numbers and calculating mentally to gain a sense of the approximate size of the answer, and have strategies to check and repeat the calculation if they are not sure whether it is right.

For these reasons schools should not normally use the calculator as part of Key Stage 1 mathematics but should emphasise oral work and mental calculation. But by the end of Key Stage 2, pupils should have the knowledge and competence to use a calculator to work out, say, (56 + 97) ÷ (133 – 85) and round the answer to one decimal place. They should also recognise that an approximate answer is 150 ÷ 50, or 3, and use this to check their calculation.

Laying the foundations for algebra

Algebra is a compact language which follows precise conventions and rules. Formal algebra does not begin until Key Stage 3 but you need to lay the foundations in Key Stages 1 and 2 by providing early algebraic activities from which later work in algebra can develop. These activities include:

- **Forming equations** When you are questioning your class you might at times ask them to give more than single word or single number answers. For example, you might sometimes expect the response to short questions such as: 'What is 16 add 8?' to be expressed as a complete statement: 'sixteen add eight equals twenty-four', which children can repeat in chorus. You might also invite a child to the board to write the same equation in symbolic form: 16 + 8 = 24.
- **Solving equations** By asking questions such as: 'Complete 3 + □ = 10' you can introduce children to the idea that a symbol can stand for an unknown number. You can also ask questions in the form: 'I double a number, then add 1, and the result is 11. What is the number?' By considering equations with two unknowns, such as □ + △ = 17, or inequalities like 1 < □ < 6, you can lead children towards the idea that the unknown is not necessarily one fixed number but may also be a variable.
- **Using inverses** Another important idea in both number and algebra is the use of an inverse to 'reverse' the effect of an operation. For example, the inverse of doubling is halving, of adding 7 is subtracting 7, and of multiplying by 6 is dividing by 6. Once they have grasped this idea, pupils can use their knowledge of an addition fact such as 4 + 7 = 11 to state a corresponding subtraction fact: 11 – 7 = 4. Similarly, pupils should be able to use their knowledge of a multiplication fact such as $9 \times 6 = 54$ to derive quickly a corresponding division fact: $54 \div 6 = 9$.
- **Identifying number patterns** Encourage children to look for and describe number patterns as accurately as they can in words and, in simple cases, to consider why the pattern happens. For example, they could explore the patterns made by multiples of 4 or 5 in a 10 by 10 tables square, or extend and describe simple number sequences such as 2, 7, 12, 17… and, where appropriate, describe and discuss how they would set about finding, say, the 20th term.
- **Expressing relationships** When discussing graphs drawn, say, in science, ask children to describe in their own words the relationships revealed: for instance, 'every time we added another 20 grams the length of the elastic increased by 6 centimetres'. They can also be asked to use and make their own simple word equations to express relationships such as:

  $$\text{cost} = \text{number} \times \text{price}$$

  By Year 6, pupils should be ready to express relationships symbolically: for example, if cakes cost 25p each then $c = 25 \times n$, where c pence is the total cost and n is the number of cakes.
- **Drawing graphs** As well as drawing graphs which display factual information, teach older pupils to draw and use graphs which show mathematical relationships, such as those of the multiplication tables, or conversions from pounds to foreign currency. Games like Battleships can be used to introduce the idea of co-ordinates to identify spaces and, later, single points. It is then possible to record graphically, for example, pairs of numbers that add up to 10.

- **Developing ideas of continuity** Another foundation stone for algebra is laid in Years 5 and 6 when you help children to appreciate that between any two decimal numbers there is always another, and that the number line is continuous. They also need to understand that quantities like heights and weights are never exact. In growing from 150 cm to 151 cm, say, every possible value in that interval has been attained because measures too are continuous.
- **Finding equivalent forms** You should emphasise from the very beginning the different ways of recording what is effectively the same thing. For example:
  - $24 = 20 + 4 = 30 - 6$;
  - $30 = 6 \times 5 = 3 \times 2 \times 5$;
  - $15 + 4 = 19$ implies that $15 = 19 - 4$, and $3 \times 4 = 12$ implies that $12 \div 3 = 4$;
  - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$ … and each of these is equivalent to 0.5 or 50%.
- **Factorising numbers** Factorising 30 as $2 \times 3 \times 5$ is a precursor of the idea of factorising in algebra. It is also a useful strategy for multiplication and division. For example, since $12 = 6 \times 2$, the product 15×12 can be calculated in two steps, first $15 \times 6 = 90$, then $90 \times 2 = 180$. Similarly, $273 \div 21$ can be worked out by using the factors of 21, first $273 \div 3 = 91$, then $91 \div 7 = 13$.

  Encourage pupils to factorise numbers as far as is possible. To factorise 24 as 6×4 is not as complete as $2 \times 2 \times 2 \times 3$.
- **Understanding the commutative, associative and distributive laws** Pupils do not need to know the names of these laws but you need to discuss the ideas thoroughly since they underpin strategies for calculation and, later on, algebraic ideas.

  Children use the commutative law when they change the order of numbers to be added or multiplied because they recognise from practical experience that, say:

  $4 + 8 = 8 + 4$ and $2 \times 7 = 7 \times 2$

  The associative law is used when numbers to be added or multiplied are regrouped without changing their order: for example,

  $(4 + 3) + 7 = 4 + (3 + 7)$ and $(9 \times 5) \times 2 = 9 \times (5 \times 2)$

  An example of the distributive law would be a strategy for calculating 99×8:

  $99 \times 8 = (100 - 1) \times 8 = 100 \times 8 - 8$

  Another example of the distributive law is a method for 'long multiplication' which prepares the way for the standard written method. In this multiplication method each part of the first number is multiplied by each part of the second, and then the products are added to find their total. So 35×24 is split up as:

	30	5
20	600	100
4	120	20

  $600 + 100 + 120 + 20 = 840$

  As well as illustrating clearly how the multiplication method works, this method provides a foundation for the later idea of multiplying out a pair of brackets:

  $(30 + 5)(20 + 4) = (30 \times 20) + (5 \times 20) + (30 \times 4) + (5 \times 4)$

Pupils who have a secure understanding of all these important ideas by the age of 11 will be in a sound position to start work on more formal algebra in Key Stage 3.

Teaching mathematics

The approach to teaching recommended by the National Numeracy Strategy is based on four key principles:

- dedicated mathematics lessons every day;
- direct teaching and interactive oral work with the whole class and groups;
- an emphasis on mental calculation;
- controlled differentiation, with all pupils engaged in mathematics relating to a common theme.

This section and the next, 'School and class organisation: some questions answered', give you, as a class teacher, some practical guidance on how to put this approach into practice.

Teaching time

To ensure that there is adequate time for developing numeracy skills, each class teacher is expected to provide a daily lesson for mathematics, which should last about 45 minutes in Key Stage 1 and 50 to 60 minutes in Key Stage 2.

It is also important to find time in other subjects for pupils to develop and apply their mathematical skills. For example, you could plan regular opportunities for measuring in science and design and technology, using properties of shapes and patterns in art, and collecting and presenting data in history, geography and ICT (see also 'Making links between mathematics and other subjects', on pages 16 and 17).

You will also need to build in time to discuss progress with individual pupils (see the section on assessment starting on page 33).

The focus on direct teaching

During each lesson you should aim to spend as much time as possible in direct teaching and questioning of the whole class, a group of pupils, or individuals.

High-quality direct teaching is oral, interactive and lively. It is not achieved by adopting a simplistic formula of 'drill and practice' and lecturing the class, or by expecting pupils to teach themselves from books. It is a two-way process in which pupils are expected to play an active part by answering questions, contributing points to discussions, and explaining and demonstrating their methods to the class.

Good direct teaching is achieved by balancing different elements:

- **Directing:** sharing your teaching objectives with the class, ensuring that pupils know what to do, and drawing attention to points over which they should take particular care, such as how a graph should be labelled, the degree of accuracy needed when making a measurement, or how work can be set out…
- **Instructing:** giving information and structuring it well: for example, describing how to multiply a three-digit number by a two-digit number, how to interpret a graph, how to develop a mathematical argument…

- **Demonstrating:** showing, describing and modelling mathematics using appropriate resources and visual displays: for example, showing how to scribe numerals, showing how to measure using a metre stick or a protractor, demonstrating on a number line how to add on by bridging through 10, using a thermometer to demonstrate the use of negative numbers…
- **Explaining and illustrating:** giving accurate, well-paced explanations, and referring to previous work or methods: for example, explaining a method of calculation and discussing why it works, giving the meaning of a mathematical term, explaining the steps in the solution to a problem, giving examples that satisfy a general statement, illustrating how the statement 7 – 3 = 4 can represent different situations…
- **Questioning and discussing:** questioning in ways which match the direction and pace of the lesson and ensure that all pupils take part (if needed, supported by apparatus or a communication aid, or by an adult who translates, signs or uses symbols), listening carefully to pupils' responses and responding constructively in order to take forward their learning, using open and closed questions, skilfully framed, adjusted and targeted to make sure that pupils of all abilities are involved and contribute to discussions, allowing pupils time to think through answers before inviting a response…
- **Consolidating:** maximising opportunities to reinforce and develop what has been taught, through a variety of activities in class and well-focused tasks to do at home, asking pupils either with a partner or as a group to reflect on and talk through a process, inviting them to expand their ideas and reasoning, or to compare and then refine their methods and ways of recording their work, getting them to think of different ways of approaching a problem, asking them to generalise or to give examples that match a general statement…
- **Evaluating pupils' responses:** identifying mistakes, using them as positive teaching points by talking about them and any misconceptions that led to them, discussing pupils' justifications of the methods or resources they have chosen, evaluating pupils' presentations of their work to the class, giving them oral feedback on their written work…
- **Summarising:** reviewing during and towards the end of a lesson the mathematics that has been taught and what pupils have learned, identifying and correcting misunderstandings, inviting pupils to present their work and picking out key points and ideas, making links to other work in mathematics and other subjects, giving pupils an insight into the next stage of their learning…

Direct teaching and good interaction are as important in group work and paired work as they are in whole-class work but organising pupils as a 'whole class' for a significant proportion of the time helps to maximise their contact with you so that every child benefits from the teaching and interaction for sustained periods.

Class organisation

From Year 1, all pupils should have a dedicated mathematics lesson every day, so that work is not mixed with other subjects as it is in an 'integrated day' approach. Direct teaching is not compatible with an integrated day. If you spent your time teaching and interacting with each group doing mathematics throughout an integrated day, there would be little time for direct teaching of any other subject.

Dedicated lessons also make it easier for you to secure a good balance between whole-class work, group teaching and individual practice, and to make the most of any support for mathematics from classroom assistants. Dedicated lessons allow you to establish routines that pupils get used to, so that you can maximise the time you spend teaching the class, as against managing it. The overall pattern of lessons will generally be the same for all classes so you will also have a common structure for developing ideas and sharing planning and teaching with other colleagues.

Teaching in Reception is discussed in question 8 of the next section (pages 26–29).

A typical lesson

A typical 45 to 60 minute lesson in Years 1 to 6 will be structured like this:

- **oral work and mental calculation** (about 5 to 10 minutes)
  whole-class work to rehearse, sharpen and develop mental and oral skills
- **the main teaching activity** (about 30 to 40 minutes)
  teaching input and pupil activities
  work as a whole class, in groups, in pairs or as individuals
- **a plenary** to round off the lesson (about 10 to 15 minutes)
  work with the whole class to sort out misconceptions and identify progress, to summarise key facts and ideas and what to remember, to make links to other work and discuss the next steps, and to set work to do at home

Oral work and mental calculation

The first 5 to 10 minutes of a lesson can be used in a variety of ways to rehearse and sharpen skills, sometimes focusing on the skills that will be needed in the main part of the lesson. On different days you might choose to do one or more of these:

- counting in steps of different sizes, including chanting as a whole class and counting round the class;
- practising mental calculations and the rapid recall of number facts in varied ways (for example, by playing an interactive number game, by giving examples of ‘a number one less than a multiple of 5’ or ‘a calculation with the answer 12’);
- figuring out new facts from known facts and explaining the strategies used;
- building on a previous strategy, and then developing it;
- identifying facts which children can learn by heart and discussing ways of remembering them;
- reviewing an activity done at home.

In this first part of the lesson you need to:

- get off to a clear start and maintain a brisk pace;
- provide a variety of short oral and mental activities throughout each week;
- prepare a good range of open and closed questions to ask the class;
- ensure that all children can see you easily and can and do take part;
- target individuals, pairs or small groups with particular questions;
- use pupils’ responses to make an informal assessment of their progress;
- brief any support staff to position themselves and give discreet help to any children who need particular support;
- avoid disruption from too much movement of pupils about the room;
- avoid running over time and move smoothly to the next part of the lesson.

The main teaching input and pupil activities

The main part of the lesson provides time for:

- introducing a new topic, consolidating previous work or extending it;
- developing vocabulary, using correct notation and terms and learning new ones;
- using and applying concepts and skills.

In this part of the lesson you need to:

- make clear to the class what they will learn;
- make links to previous lessons, or to work in other subjects;
- tell pupils what work they will do, how long it should take, what, if anything, they need to prepare for the plenary session and how they are to present it;
- maintain pace and give pupils a deadline for completing their work.

When you are working directly with the **whole class** you need to:

- demonstrate and explain using a board, flip chart, computer or OHP;
- involve pupils interactively through carefully planned questioning;
- ensure that pupils with particular learning needs in mathematics are supported effectively with appropriate resources and wall displays, and adult help;
- identify and correct any misunderstandings or forgotten ideas, using mistakes as positive teaching points;
- highlight the meaning of any new vocabulary, notation or terms, and get pupils to repeat these and use them in their discussions and written work;
- ask pupils to offer their methods and solutions to the whole class for discussion.

When you are working directly with **groups** you need to:

- have a manageable number of groups (usually a maximum of four), so that you know what each group should be doing at any time;
- decide how groups will be introduced to tasks and how the group work will end;
- control the degree of differentiation (for example, provide tasks on the same theme and usually at no more than three levels of difficulty);
- provide activities, tasks and resources that don't involve children in a long wait for turns and which keep them all interested, motivated and on-task;
- sit and work intensively with one or two of the groups, not flit between them all;
- brief any support staff or adult helpers about their role, making sure that they have plenty to do with the pupils they are assisting and will not interrupt you;
- avoid interruption by pupils by making sure that those working independently in a group know where to find further resources, what to do before asking you for help and what to do if they finish early.

When you are providing work for **individuals or pairs** you need to:

- keep the class working on related activities, exercises or problems;
- target individuals or pairs for particular questioning and support;
- during paired work, encourage discussion and co-operation between pupils.

The plenary session

The plenary is an important part of the lesson. It is a time when you can help pupils to assess their developing knowledge and skills against any targets they have been set and to see for themselves the progress they are making. It is also a time when you can relate mathematics to their work in other subjects: for example, how their work on calculation will be used in science, or how their measuring skills will be practised in physical education.

For example, this part of the lesson can be used to:

- ask pupils to present and explain their work, or mark a written exercise done individually during the lesson, so that you can question pupils about it, assess it informally and rectify any misconceptions or errors;
- discuss and compare the efficiency of pupils' different methods of calculation;
- help pupils to generalise a rule from examples generated by different groups, pairs or individuals;
- draw together what has been learned, reflect on what was important about the lesson, summarise key facts, ideas and vocabulary, and what needs to be remembered;
- discuss the problems that can be solved using the ideas and skills that have been learned;
- make links to other work and discuss briefly what the class will go on to do next;
- remind pupils about their personal targets and highlight the progress made;
- provide tasks for pupils to do at home to extend or consolidate their class work.

In this part of the lesson you need to:

- have a clear idea of the purpose of the plenary session and what you want to achieve in it;
- make sure that the main part of the lesson does not over-run, so that there is enough time for the plenary;
- plan carefully how pupils are to present their work, if they are to do this, and how long it will take;
- bring the lesson to a close and evaluate its success.

The outline structure of a typical lesson should not be seen as a mechanistic recipe to be followed. You should use your professional judgement to determine the activities, timing and organisation of each part of the lesson to suit its objectives.

In the main part of the lesson, in particular, there is scope for considerable variety and creativity, with a different mix of work with the whole class, groups, pairs and individuals on different days, although each lesson should include direct teaching and interaction with the pupils, and activities or exercises that pupils do. Overall, there should be a high proportion of work with the whole class but there may be more in some lessons than in others. For example, at the start of a new unit of work you might need more time for explanation and discussion with everyone together for the whole lesson, and the plenary may be very short. On the other hand, where you have identified general errors or misunderstanding during the main part of a lesson, you might need a longer plenary to sort them out. At the end of a unit of work it can be useful to use the plenary to look back with the whole class over a number of lessons to draw together what has been learned and to identify the key points and methods that you want pupils to remember and use in the future. For this kind of plenary session, you may need a much longer time than usual.

Out-of-class work and homework

Your daily mathematics lessons provide opportunities for children to practise and consolidate their skills and knowledge, to develop and extend their techniques and strategies, and to prepare for their future learning. You can extend these opportunities through out-of-class activities or homework.

Not all out-of-class work needs to be written work which then has to be marked. You can equally well ask your class to:

- do an activity which makes use of the home context, such as tipping out a purse and counting what is in it, or weighing things on the kitchen or bathroom scales;
- play a number game or work on a number puzzle;
- learn some number facts or multiplication tables by heart;
- gather information to use in the next lesson: for example, collect data or take measurements;
- think about how they might solve a problem;
- prepare their contribution to a group presentation to the class.

For older children, work outside the normal lesson can be completing a short written exercise or task which consolidates and develops from work done in class, with modifications of the presentation for any children who need them (see question 2 in 'School and class organisation: some questions answered', pages 19 and 20). You then need to mark the work promptly and thoroughly so you can give children some feedback on their progress. For example, an exercise at the end of a unit of work, or a few days after it, and which pupils do independently, can give you useful diagnostic information on who has learned what and who needs extra support. Sometimes you might set a quiz with a mix of short questions which you expect children to do quickly and successfully. At the start of the next lesson you can read out answers and children can mark their own work. You should then go through any questions that proved to be difficult.

Out-of-class activities need to be frequent, short and focused. They should be varied, interesting and fun so that they motivate children, stimulate their learning and foster different study skills.

But whatever work you set, you should give children feedback to show them that their work is important and their efforts are valued. You should also indicate whether and how their work might be improved. For example, you might discuss a problem briefly in the plenary part of a lesson and ask the children to tackle it in preparation for the next lesson. This could start with sharing and refining methods and solutions, which are then used to inform the main teaching activity, when you give similar or linked problems to each group. Or you could use the plenary to introduce a game which helps children to practise the recall of number facts and which they can play with their families or friends. In the mental and oral work at the start of the next lesson you could focus on the recall of these facts, so that you can see through your interactions with the children which of them have good recall and which need some additional support.

Making links between mathematics and other subjects

You need to look for opportunities for drawing mathematical experience out of a wide range of children's activities. Mathematics contributes to many subjects of the primary curriculum, often in practical ways. Activities such as recording the growth of a plant or an animal, measuring temperature and rainfall, or investigating the cog wheels in a bicycle can provide data or starting points for discussion in your mathematics lessons as well as opportunities to apply and use mathematics in real contexts.

English Mathematics lessons can help to develop and support pupils' literacy skills: for example, by teaching mathematical vocabulary and technical terms, by asking children to read and interpret problems to identify the mathematical content, and by encouraging them to explain, argue and present their conclusions to others. Equally, the literacy hour can support your daily mathematics lesson. For example, in Reception and Key Stage 1, stories, rhymes and songs can be chosen which rely for their appeal on the pleasure of counting, the sequencing of events, and the use of everyday words such as 'on' and 'under', 'up' and 'down' to describe position or direction. In Key Stage 2, the literacy hour can be used to read non-fiction in which mathematical vocabulary, graphs, charts and tables have to be interpreted.

Science Almost every scientific investigation or experiment is likely to require one or more of the mathematical skills of classifying, counting, measuring, calculating, estimating, and recording in tables and graphs. In science pupils will, for example, order numbers, including decimals, calculate simple means and percentages, use negative numbers when taking temperatures, decide whether it is more appropriate to use a line graph or bar chart, and plot, interpret and predict from graphs.

Art, design and technology Measurements are often needed in art and design and technology. Many patterns and constructions are based on spatial ideas and properties of shapes, including symmetry. Designs may need enlarging or reducing, introducing ideas of multiplication and ratio. When food is prepared a great deal of measurement occurs, including working out times and calculating cost; this may not be straightforward if only part of a packet of ingredients has been used.

Information and communications technology Children will apply and use mathematics in a variety of ways when they solve problems using ICT. For example, they will collect and classify data, enter it into data handling software, produce graphs and tables, and interpret and explain their results. Their work in control includes the measurement of distance and angle, using uniform non-standard then standard measures. When they use computer models and simulations they will draw on their abilities to manipulate numbers and identify patterns and relationships.

History, geography and religious education In history and geography children will collect data by counting and measuring and make use of measurements of many kinds. The study of maps includes the use of co-ordinates and ideas of angle, direction, position, scale and ratio. The pattern of the days of the week, the calendar and recurring annual festivals all have a mathematical basis. For older children historical ideas require understanding of the passage of time, which can be illustrated on a time line, similar to the number line that they already know.

Physical education and music Athletic activities require measurement of height, distance and time, while ideas of counting, time, symmetry, movement, position and direction are used extensively in music, dance, gymnastics and ball games.

The key to making the most of all these opportunities is to identify the mathematical possibilities across the curriculum at the planning stage. You should also draw children's attention to the links between subjects by talking frequently about them, both in mathematics and in other lessons.

School and class organisation: some questions answered

1 Can we plan work for groups of children?

All pupils gain from working in groups, in pairs or as individuals from time to time.

Whether you have group work may depend on where you are in a series of lessons. For example, you might introduce a new unit of work with a main teaching activity that is mostly with the whole class while you explain, demonstrate, ask questions and discuss answers. Short tasks for pupils to do, perhaps in pairs, and a short written exercise that children tackle individually, may also be appropriate in this lesson. Another possibility when you introduce a new unit of work is to begin the main part of the lesson with some teaching input to the whole class, then to start the more confident pupils on an activity or exercise while you continue to teach the rest. When they too are ready to work independently, set them going while you support the one or two pupils who are likely to have the most difficulty with the task.

In the next couple of lessons, the main activity might consist of group work on the same theme, although your lesson will still begin and end with the whole class. Occasionally, group work should allow pupils at all levels of attainment to work with each other on an equal footing, but grouping pupils by attainment – perhaps two groups in the middle range, one of higher achievers and one of those who find mathematics more difficult – allows for a controlled degree of differentiated work on the topic being taught to the whole class, with a simplified task for some pupils and a harder challenge for others.

If, for example, you have four groups, you can teach two of them during a lesson for about 15 minutes each, giving positive direction and guidance; in the next lesson you can teach the other two groups. In the groups working independently children can, of course, work on the group task as individuals or in smaller groups of up to four. In your plenary, if different groups are giving feedback, they have a common interest since they have all been working on the same topic, albeit at different levels of difficulty. You can draw together ideas that all pupils have worked on and make an informal assessment of their understanding to help you plan the next lesson.

The final lesson of a unit of work might centre around one or more open-ended problems, games or puzzles for the whole class which allow responses at different levels. After you have introduced an activity to everyone, children can continue it in pairs. During the paired work you can support and teach particular pairs or individuals you wish to target, such as pupils who have been absent.

You need to prepare your class for these ways of working, so that pupils and adult helpers don't interrupt you when you are teaching a group. Pupils need to know how their lesson time is to be used and what routines they should follow when they are working independently before they ask an adult for help: for example, how to collect and return any resources that they need, what to do if they have finished something or are 'stuck', and so on. There may also be implications for how you arrange tables and chairs (see question 9, page 29).

2 Are there other ways of providing differentiated work?

The children in each class should, as far as possible, work together through the year's programme described in the Framework, so that all children participate when a new unit of work starts and can take part in the plenary. When classes taking part in the National Numeracy Project were taught in this way, all groups of pupils made significant progress, including the most able, but those who made more progress relative to other groups were the pupils who initially had achieved the lowest test scores.

There are several ways in which the needs of particular pupils can be met, partly through the differentiated group work and open-ended tasks described under question 1, and partly through other teaching strategies.

Differentiation during whole-class oral work

For your daily session of oral work and mental calculation with the whole class, think about the questions you will ask, planning some with particular children in mind. Your first few questions might be at a level that all children can manage, to get them involved and interested. When children are counting round the class, for example, you might point to the child who should say the next number; for the smaller, easier numbers point to those who tend to struggle and leave larger, harder numbers for higher achievers. When you are directing questions to the whole class, build in enough 'wait time' for all children to think before expecting the class as a whole to answer your questions. This benefits everyone. Ask the whole class to repeat a correct answer together using a complete sentence, so that all pupils, including those learning English as an additional language or with special educational needs, get to say larger numbers aloud and use mathematical vocabulary correctly.

You can also use open questions that allow all children to take part: for example, 'What numbers can you make using each of the digits 2, 3 and 4 once, and any operations?' Encourage the class to discuss their answers in pairs before they give you a response. You can also target individuals or groups: 'Louise, can you make 9?' 'Winston, can you make 17?' 'Can this group make 5 in three different ways?'

Even closed questions such as 'What is 30×25?' can be opened up by discussing the methods used. For example, some pupils may first do 3×25 and then multiply the result by 10, others may do 30×20 first and then add 30×5, while others may multiply 30 by 100 then divide the result by 4. Some may be helped by jotting down the interim stages of these calculations, while some will be able to do the entire calculation mentally. The answer $30 \times 25 = 750$ can lead to discussion of other statements that can now be deduced, such as 3×25, or 300×25, or $750 \div 30$, …

It is important that all pupils can take part in the discussion. Make sure that they can all see or hear what is being done, and can respond orally, or through number cards, symbols or tactile materials, or with support from an adult.

Differentiation during written work or homework

There are several ways in which the presentation of written work or homework can be adapted to suit particular needs, without varying either the task or the level of difficulty. For example, it can be presented on an audio tape rather than a written page, or in enlarged print or by using tactile materials. For some children all that is

needed is simplified vocabulary, or extra diagrams or illustrations to illuminate key points. Others may need the task broken down into a series of guided steps. There may also be children who continue to need apparatus to support their thinking while others manage without. For instance, with the addition and subtraction of larger numbers, some children will continue to use a number line, while others will already be able to do the calculations mentally. The freedom for children to choose their own support materials is therefore important, and you should ensure that suitable resources are on hand. At the same time you need to encourage them to dispense with the apparatus when you judge that they are ready.

Some children work faster than others, perhaps because they use short-cuts, or are generally more confident and more able. They may need to do fewer examples and be moved on to extension or enrichment tasks linked to the theme of the lesson so that they use and apply their skills in more challenging contexts, including those which ICT can offer. Others may need longer to practise and consolidate what they have been learning. For them, problems in which they use and apply skills, and the use of ICT, are also an important part of their learning, but the problems may need to be simpler and sometimes left until later in a unit of work, or when a unit is revisited, or in extra time outside the lesson.

3 Should we set the pupils for mathematics lessons?

Larger schools with parallel classes sometimes deal with a range of attainment by organising 'ability sets' for mathematics lessons. The advantage is that planning can be easier if the attainment gap in a class is not too wide. It is also possible to set across, say, Years 5 and 6, if both years are timetabled for their mathematics lessons at the same time, although you need to ensure that when the Year 5 pupils move into Year 6 they do not simply repeat the previous year's activities.

Any setting arrangements need to be flexible to allow easy transfer of pupils between sets. On its own, setting does not necessarily help to close the attainment gap across the year group over time. Children are quick to spot and interpret the significance of this kind of provision.

The success of setting depends on very careful monitoring, close team work and co-operative planning among staff to make sure that expectations for all pupils are suitably high and that lower expectations are not justified simply because pupils are in a 'lower set'. Governors and parents need to know why classes are set for mathematics, especially if pupils are not taught in sets for other subjects.

As standards improve over time, the range of attainment in each year group ought to reduce so that it becomes possible to cater for the diversity of needs by grouping pupils within the same class.

4 How can we cater for pupils who are very able?

Nearly all able pupils will be taught with their own class, whether it be a higher-ability set or a mixed-ability class. They can be stretched through differentiated group work, harder problems for homework, and extra challenges – including investigations using ICT – which they do towards the end of a unit of work when other pupils are doing consolidation exercises.

When you are working with the whole class, you can direct some questions towards the most able pupils, just as you can direct some specifically towards the children who find mathematics most difficult.

The yearly teaching programmes described in the Framework leave about one week in each term unallocated, when pupils who are very able can, for example, carry out a sustained mathematical investigation and continue it at home. There are many good publications to support this kind of work and schools with access to the Internet can also download suitable material from a problem-solving web-site.

Pupils who are exceptionally gifted in many subjects, and who are sufficiently mature, are sometimes promoted to work with an older age group. They are able to deal with abstract mathematics much earlier than other children and, for them, some acceleration is desirable. Very occasionally, a pupil is exceptionally gifted in mathematics, but not in other subjects. Special arrangements are sometimes made for these pupils. For example, by timetabling Year 3 and Year 4 mathematics lessons at the same time, an exceptionally gifted pupil in Year 3 can be taught the subject with the Year 4 class and benefit from discussion with other pupils working at a similar level. Where this is not possible, exceptionally gifted pupils can follow individualised programmes at appropriate times in the daily mathematics lesson, with far fewer practice examples and many more challenging problems to tackle.

5 How do we cater for pupils with particular needs?

The daily mathematics lesson is appropriate for almost all pupils. You should aim to ensure that everyone makes progress and gains positively from the lesson, and to plan lessons so that all pupils can be included, rather than over-differentiate.

Individual needs do not necessarily warrant individual attention. The needs of pupils regarded as 'special' are not essentially different from those of other children. Instead of focusing on differences, you might emphasise the links with the needs of all learners, and use them productively to improve learning opportunities for all children. The key things are to give greater emphasis to children's language during mathematics lessons and to encourage all pupils to learn by working together.

All children can benefit from oral work. In whole-class sessions, plan some questions specifically for pupils at the earlier stages of learning English as an additional language (EAL), and others for those with special educational needs (SEN), and ask named children to respond. At times you may want to place those with particular needs close to you, so that they can be given some discreet help: for example, when children discuss an answer in pairs before responding. Pupils with hearing impairments may also need to sit closer to you, or be helped to take part in the activity through signing or support given by another adult.

Reading difficulties or lack of familiarity with English can slow some children's progress with mathematics. Use flash cards and illustrated wall displays to show the specific mathematical vocabulary for a unit of work. Minimise any written instructions and explanations on worksheets and written exercises. Remember that mathematics has a strong visual element and capitalise on this wherever you can to illuminate meaning. Make frequent use of a number line, 100 square, number apparatus, pictures, diagrams, graphs, computer programs... and games and puzzles where the rules are picked up quickly by watching a demonstration.

Teachers can involve and support all pupils through the strategies for differentiation described under questions 1 and 2 (see pages 18–20). Beyond this, the assumption is that appropriate support will be dedicated to help those pupils who need it to make progress at the appropriate rate. You should take quick action to help them, as it is much easier to catch children early on than to struggle later with a backlog of problems. Use the dedicated time each half term to assess and review children's progress and make sure that the key learning objectives are secure. Use the week of unallocated teaching time each term for extra consolidation for pupils who need it, focusing on the misconceptions or weaknesses you have identified. You could also aim to recruit parents to help their children in specific ways.

Where schools are able to offer lunch-time or after-school activities, teachers or other adults might offer extra support so that particular pupils can either be prepared for or consolidate their learning from the daily lesson. For example, pupils with hearing impairments, or EAL pupils, could be introduced to the new vocabulary they will meet in the next week's oral work; mentoring sessions may be needed for children whose behaviour causes concern; others may benefit from practising skills and making individual use of suitable computer software.

Other points related to pupils learning English as an additional language

Take care not to underestimate what children can do mathematically simply because they are new learners of the English language. The expectation should be that they progress in their mathematical learning at the same rate as other pupils of their age. Whole-class sessions can provide helpful adult models of spoken English, and opportunities for careful listening, oral exchange and supportive, shared repetition. Group work provides opportunities for intensive, focused teaching input. You will probably need to repeat instructions for EAL pupils and to speak more clearly, emphasising key words, particularly when you are describing tasks that they are to do independently. Put picture cues on worksheets or puzzle cards and simplify the words used, but not the mathematics (except where an EAL pupil also has special educational needs that warrant this).

Peer-group talk is important in helping children to make sense of and apply mathematical ideas. It helps if English language beginners can converse with other children or adults who speak the same home language when they are doing practical group activities, playing mathematical games or working with a computer.

Don't ask individual children who are in the early stages of learning English as an additional language to present their work orally before they are ready. Allow them time to watch and listen to those fluent in English explaining and demonstrating their methods to the class using a board or flip chart. Invite them too to demonstrate, on the board or with apparatus – they will often show capabilities that are as good if not better than their peers – but without any pressure to accompany their demonstration with an oral explanation in English before they are ready.

Encourage them to join in things that all children do in chorus: counting, reading aloud whole number sentences, chanting, finger games, songs about numbers, and so on. The structure of rhymes, and the natural rhythm in songs and poems, play an important part in developing number sense in any culture. Try to use stories and rhymes that will support access to meaning for pupils from a range of cultural backgrounds. As soon as English language beginners are reasonably confident at saying something together with others, ask them to say it again on their own. Give

them plenty of time and much praise for doing so, and also check for understanding.

Emphasise the importance early on of children learning to understand, say, read and write numbers in English, also signs such as plus (+) and minus (–) and words such as 'add' and 'subtract'. They may well be familiar with the meanings of such words in their home language. Get them to show the class the ways that numbers are spoken and written in their home language, and to demonstrate any board games or playground games that they know and which involve numbers. Children who speak and read only English are often fascinated by the similarities with the number system that they are familiar with and they too gain from the experience.

Pupils with special educational needs and individual education plans

All teachers will have in their class some children whose progress warrants special consideration. Their difficulties may have physical, sensory, behavioural, emotional or neurological causes, or may stem from a legacy of poor learning that inhibits their current learning. Some of the one in six pupils identified by primary schools as having special educational needs may have problems with mathematics, often but not always in association with literacy problems caused by difficulties of varying degrees of complexity. It is not possible in this document to give detailed advice covering every type of special educational need, but as a general guide you should aim to include all these pupils fully in your daily mathematics lesson, so that they benefit from the emphasis on oral and mental work and participating in watching and listening to other children demonstrating and explaining their methods.

For example, there is no good reason why pupils with physical or sensory disabilities should work on any sort of separate programme. For most of them it is simply a question of access, and materials, equipment and furniture should be adapted to meet their particular needs so that they can work alongside their peers. They need to work on the objectives for their year if these are within their grasp, with the emphasis on access and support. Adaptations that may be necessary are, for example, preparation for the oral and mental part of the lesson and the pace at which it is conducted, the use of signing, Braille and symbols, and the provision of a range of tactile materials, technological aids and adapted measuring equipment.

Pupils with emotional or behavioural difficulties can benefit from the structure and routines of the daily mathematics lesson. Adaptations are usually needed during the main teaching activity, with a shorter time for independent group work if adult support is not available. Learning to work independently with increasing self-confidence is important for these pupils but it has to be introduced slowly, cultivated deliberately and rewarded. Tasks and timings are critical; you need to consider them carefully if you are to maintain the pupils' motivation and interest.

Where pupils' learning difficulties extend to mathematics, use the Framework to identify suitable objectives to be incorporated into individual education plans (IEPs), tracking back to earlier stages if it is appropriate to do so. Keep these objectives in mind when you are planning group work, when you can address special needs through simplified or modified tasks and the use of support staff to consolidate key points. Where appropriate, you could develop a more manageable 'group education plan' with common objectives and learning targets for a group.

A pupil whose difficulties are severe or complex may need to be supported with an individualised programme in the main part of a lesson. Consult any staff who are

giving support to the pupil, then use your judgement to decide whether this is appropriate, bearing in mind the lesson's aims and the short-term aims in the IEP.

Spell out in all pupils' IEPs not only who will provide adult support but when and how it will be used. For example, children with hearing impairments may need an adult to prepare them for whole-class oral work and sign for them during it, or those with learning difficulties may need to be guided during group work in the use of apparatus to support their thinking. IEPs which address behavioural issues can set useful objectives about appropriate behaviour in independent group time.

How can the Framework be used in special schools?

Many of the Framework's principles are applicable to special schools, such as planning from clear teaching objectives, with an emphasis in lessons on oral work and mental calculation, visual interest, involvement and interaction, and keeping pupils working together as far as possible. Special schools are encouraged to adopt the Framework but should also adapt arrangements to suit their particular circumstances.

For example, the notion of 'whole-class teaching' can be modified, as it is different when there are 10 to 14 pupils in a class, and the teacher is at times supported by other staff. There may be times when all the pupils are taught together for their daily mathematics lesson, just as in a mainstream school, but others when two 'whole-class lessons' are taking place in the same room, with the class teacher teaching one half of the class, and a support assistant working with the other half.

It is possible that in some special schools all or nearly all of the pupils in a class have learning difficulties that extend to mathematics. In this case it may be best to base the work for Key Stages 1 and 2 on the teaching programmes for Reception and Years 1 to 2, taking two years to cover what will be taught in one year in a mainstream school. Extra 'small steps' can be inserted, and contexts for practical work and problem-solving adapted for the pupils' ages. There will then be plenty of time for consolidation without sacrificing the breadth of the teaching programmes or the principle of planning from clearly defined objectives. The routine of the daily mathematics lesson is best built up over a period of weeks. Aim first to establish the daily oral and mental work. Then introduce routines for the main part of the lesson and the plenary, concentrating on a strong oral or communication technique that promotes interaction, combined with a good range of practical activities.

Similar principles apply where pupils have severe learning difficulties. The pupils may first need to work at levels earlier than those in the Framework, and then have further extensions to the pace at which the yearly teaching programmes are covered and to the extra 'small steps'.

6 How is it best to use classroom assistants, including support for SEN and EAL?

The role of support staff is to help you make sure that each child plays a full part in every lesson. You should give them copies of this Framework and involve them in planning lessons. You will need to brief them very thoroughly about each lesson and their particular role in it. Make sure that they know not only what children are to do but what children are to learn. Draw their attention to the booklet *Mathematical*

Vocabulary (see page 3) and identify the words they might focus on in a particular unit of work.

During any whole-class oral work, ask support staff to position themselves close to any children who need special help and provide this discreetly by, for example:

- prompting shy or reticent pupils;
- signing or translating core vocabulary or phrases;
- helping children to use specific individual resources to find an answer, such as personal number cards or table-top number lines;
- operating individualised ICT resources as indicated in children's IEPs.

They should also observe carefully the responses of the children they will be working with later in the lesson to inform the support they will give.

If you have a general classroom assistant and are organising group work with four groups in the main part of the lesson, explain that you will work with two of the groups, and that the assistant should oversee the other two. Ask the assistant to:

- ensure that children interpret instructions correctly, concentrate and behave responsibly;
- remind children of teaching points made earlier in the lesson;
- question children and encourage their participation (you will need to suggest the questions and prompts that would be appropriate, and any particular children whom they should focus on);
- look for and note any common difficulties that children have, or mistakes that they make, so that you can address these in the plenary and in future lessons;
- use and make available to children a number line and/or 100 square, visual or practical aids, or a computer with suitable software, especially when they are helping children with difficulties or misunderstandings.

Include assistants and other adult helpers in whole-school training days and draw their attention to the training offered through the specialist teacher assistant (STA) scheme, if your LEA is taking part in this.

7 How do we organise and plan for mixed-age classes?

Often mixed-age classes are unavoidable because of the size of the school but if there are parallel mixed-age classes in your school you could try to reorganise them for mathematics lessons into year groups or even 'ability sets'. This helps to reduce the attainment gap in each class so that planning is easier for you.

Classes of children from two year groups can be taught in the same way as classes with a single year group. The Framework is designed to make sure that during the main part of the lesson all the children in the class can work on the same topic at the same time, if necessary at different levels during differentiated group work. You can use the termly planning grid for either age group, since there are relatively few changes in the balance of topics from one year to the next, or you can design your own, but choose objectives from the teaching programmes for each of the two years.

When you are questioning the class as a whole, include some questions targeted at individual children, sometimes the younger ones and sometimes the older. When you are organising group work, say with a Year 3 and Year 4 class, aim for four

groups, one upper and one middle group for Year 4, and one middle group and one lower group for Year 3. The two middle groups can usually do the same work, so that you still plan activities at three levels. In the following year, when the Year 4 pupils have left the class, the two Year 3 groups are promoted to form a new upper group and one of the middle groups. In this way, they don't repeat activities when you revisit units of work.

Very small schools

In very small schools with three or more year groups in the same class, it is still possible to have a profitable oral and mental starter with the whole class. This can focus on counting and practising mental strategies previously taught to most of the class. You should use the full range of strategies for differentiation described under question 2 (pages 19–20), including open-ended examples that encourage the rehearsal of known number facts, such as: 'Give me two numbers which make …'

When you introduce a new topic in the main part of the lesson, further work with the whole class is sometimes possible. If the range of attainment is too wide, the class can be divided into two for direct teaching purposes. However, older pupils benefit from explaining what they know already about a topic to younger ones, with prompts from the teacher. For them, the whole-class session is useful revision and consolidation. For the younger children the work will be new and demanding, and should be reinforced by further direct teaching. While you do this with the group of younger children, the middle age group can work on an introductory activity. The older, higher-attaining pupils can continue with consolidation exercises based on their previous work on that topic. On the next day you can introduce the older pupils to more advanced work, and then the middle group.

Children in classes of three or more year groups are often allocated to a group according to the activity which is suitable for them rather than their age. It is better to aim for no more than four groups and, as in larger schools, all children should be able to work independently on tasks when you are working with another group. Pupils with special needs may need some adaptations to their group activity, especially if they find recording difficult.

For the plenary, younger children can explain the hardest examples they had to do. Older pupils can be challenged to explain the easiest examples they worked on but in such a way that the youngest pupils can understand. As in other schools, you should establish clearly in the plenary what has been learned, and identify and put right any misunderstandings. Where necessary, hold the plenary with half the class on alternate days. If one half has received most or all of the direct teaching in a particular lesson, then your attention can be given to the other half in the plenary.

The key is flexibility and capitalising on what you have learned from the Literacy Strategy. You need to make arrangements which work for your particular situation, bearing in mind the need for regular opportunities for oral and mental work and sustained direct teaching for all children.

8 How can we work in Reception?

Some children will be just four when they start in Reception in September. Others will start in January or April at rising five and will complete only one or two terms in the class before moving to Year 1. Some will have been to nurseries or playgroups

and some will not. Teaching needs to include a wide range of techniques to ensure effective learning for this range.

A Reception class is typically organised to promote the social skills and developing mathematical understanding of young children through stories, songs, rhymes and finger games, board games, sand and water, construction on a large and small scale, imaginative play, outdoor play and 'playground' games, cooking and shopping, two- and three-dimensional creative work with a range of materials, and by observing numbers and patterns in the environment and daily routines.

Given this organisation, your daily mathematics lesson can be planned like this:

- an introduction with the whole class, usually involving some counting, with finger games, number rhymes and songs;
- some teaching of the whole class on the main mathematics topic for the day;
- group activities:

  either for everyone in small groups simultaneously: for example, in an outdoor lesson with skittles, bean bags, hoops… with scoring built into the activities, or in an indoor lesson with shapes to be made from different media;

  or one or more play activities, linked to the theme of the lesson, worked on by groups in turn during the day, usually supported by an adult: for example, exploring 'one more' when buying stamps in the class post office, when finger painting, when making jumps in outdoor play;
- a plenary with the whole class after the group activities are ended, to consolidate and extend through discussion and questioning what they have been learning and to praise progress.

Your aim should be to prepare children, by the end of Reception, for the dedicated mathematics lesson of about 45 minutes that will be part of each day in Year 1. For example, you will need to help them to learn how to listen, how to show and talk about what they have been doing in front of other children, how to find and use the equipment that they need, how to take turns, and so on.

What teaching approaches are appropriate in Reception?

If you base your teaching on the objectives in the Framework for Reception, you can feel confident that you are working towards the Early Learning Goals and preparing children for starting the National Curriculum in Year 1. You can then plan interesting, linked activities and talking points with your chosen objectives in mind. In this way, your teaching is focused on the mathematics and is not left to chance.

On some days you can introduce mathematical ideas because they are interesting and worthwhile in themselves. For example, you might ask all the children to play a game to identify shapes, or a dice game which involves counting or adding. A story such as *The Tiger Who Came to Tea* or *Kipper's Toy Box* could be the focus for a class lesson on counting. Or you might go outside to teach about shape and space by getting children to explore the shapes they make with their bodies, and the space they use in moving. In these lessons, mathematics is the particular focus, and all children work on it at the same time.

On other days you can plan opportunities for learning in contexts which children find relevant and interesting. For example, you might put objects to encourage counting in the sand and water trays, such as speckled frogs, a lily pad and a 'pool'.

An opportunity like this can be introduced to the whole class. In this example, it may be by singing and acting different versions of *Five Little Speckled Frogs*, then showing the frogs in the sand and water trays. After the introduction, small groups can be scheduled to play freely in the 'opportunity' area but you or an assistant need to intervene in the play to question the children and develop their understanding in ways that you have planned in advance.

Ordinary classroom routines such as taking the register, changing for PE or lining up are also good opportunities for counting and reinforcing mathematical ideas. Objects that children bring to school can trigger discussions about numbers or shapes. Capitalise on such opportunities wherever you can to supplement your 'daily mathematics lesson', linking the ideas to other work the children have done.

Where can we start with Reception children?

The supplement of examples for Reception illustrates what a child who reaches the age of 5 in the autumn term, and who spends a full year in the class, should know and be able to do. Variations of knowledge and experience for those who spend less time in the class are kept in mind by the use of expressions such as:

'Begin to…' 'Show awareness of…' 'Start to use in a practical context…'

and by regular revisiting of topics term by term to absorb children who join the class throughout the year.

Because their pre-school experiences are so varied, and because their ages are so relatively different, children will start school with very mixed experiences and understanding. It is better to find out about and build on the awareness children already have than to start with an assumption of a lack of knowledge. Most children will have their own personal experience of numbers, such as the ages of people in their family, their door number, bus number, telephone number, family lottery numbers, the time of their favourite TV programme and their bed time, and so on.

Records from nurseries and playgroups can provide information about the children's performance in relation to the Early Learning Goals. Your school will also have baseline assessments to give you information about each child's starting points. All this is important information on which to build your planning.

You should also listen to what parents tell you about their child's mathematical progress. They are often anxious to know how to help. Keep them fully informed about how their children are being taught and suggest mathematical games and activities that they can play with their child at home. Make sure that each family has copies of *Count and Figure it out Together* distributed by the Basic Skills Agency.

Although there may be differences in the capabilities of a child who has spent a year in Reception and who is nearly six, and one who is just five and has been there for only one term, these differences usually lessen gradually during Key Stage 1.

Forging links with Year 1

Children in Year 1 follow the National Curriculum. Some Reception children may be under statutory school age so the emphasis of the Framework for Reception is different from that for Year 1. It is expected that children in Reception will receive some direct teaching and talk about mathematical ideas, and will explore those ideas through structured play and practical work, sometimes recording informally what they have done with objects or drawings. The main emphasis is on different

aspects of counting, such as knowing the number names, putting numbers in order, counting for a purpose, adding and subtracting by counting on and counting back, and so on, with little or no 'reading' or 'writing', other than learning to recognise and trace the first few numerals.

Summer-born children may start school in Year 1 without spending any time in Reception. Year 1 teachers can use the Framework for Reception to inform their teaching of these and any other children who have not had a complete year in a Reception class. Equally, Reception teachers and assistants should refer regularly to the Framework for Year 1 so that they are clear about where the work is heading. This should help to ensure that all children have made a sound start on the National Curriculum for mathematics by the end of Year 1.

9 How might we arrange the furniture?

How you arrange the furniture will depend on the size and shape of your room but each child needs to be able to see you, the board and their table top easily when they are seated, for both whole-class and group work. Seating arrangements for mathematics don't need to dictate the arrangements for other subjects. In mainstream classes there are children who are capable of moving the tables and chairs and they soon get used to a brisk routine of doing so.

- One solution is to arrange clumps of three or four tables in rectangles, with the narrow ends of the rectangles towards the front. Six to eight pupils can then sit in a U-shape round three sides of the rectangle so that no child has his or her back to the front. When you sit down to teach the group you can work from the vacant short edge, so aim to keep a spare chair there.
- Another solution is to arrange the tables in one large U-shape. This has the advantage that all the pupils can see each other as well as you, and the central area can be used for floor demonstrations. Or you could arrange one U-shape within another, with the inner U reserved for those who might need to sit closer to you or the board, or who showed misunderstandings in a previous lesson.

It is not essential for a class of any age to sit in a carpeted area for part of their mathematics lessons. They can just as easily sit at tables. Bear this in mind when you are organising your classroom, especially if it is small. If you do teach in a carpeted area, make sure that a board or flip chart is available on which you and the class can demonstrate and explain, and that there is enough space for pupils to do so without climbing over others. Make sure too that any pupils whose special needs warrant that they sit in particular positions are well placed. Where necessary an assistant should give discreet support by using a small, hand-held white board to mirror the work on the main board.

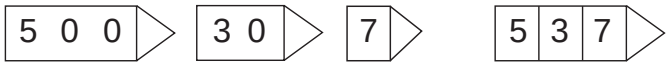
10 What resources do we need?

Beside a board, each classroom should have a large, long **number line** for teaching purposes, perhaps below the board, and at a level at which you and the children can touch it. A 'washing line' of numbers strung across the room, and which can be added to or altered, is useful. Provide table-top number lines, marked and unmarked, for individual use. For Reception and Year 1, **number tracks** with the spaces numbered to 20, rather than number lines with the points numbered,

are helpful, including those made from carpet tiles. You could also have a floor 'snake' for children to move along in corridors, the hall and the playground. For Year 2, lines need to extend to 100; by Year 4 they should include negative numbers; Years 5 and 6 need marked and unmarked lines on which decimals and fractions can be placed.

Equip each child with their own pack of **digit cards** 0 to 9 to hold up when answering questions in a whole-class setting. Two-digit numbers can be formed from cards held side-by-side. Younger children can use their fingers, both for counting and for showing answers to questions.

Place value cards are equally useful. With nine cards printed with multiples of 100 from 100 to 900, nine with multiples of 10 from 10 to 90, and ten with the numbers 0 to 9, two- or three-digit numbers can be built up by overlapping cards of different widths.



Also useful are bundles of **addition and subtraction cards** for number bonds, first bonds to 5, then to 10, extending to 20. Children can use these, for example, to find all the cards with an answer of 7, setting them out systematically. They can also be used to form equations or inequations: for example, 3 + 7 = 2 + 8, or 5 + 4 > 10 – 2. **Symbol cards** for +, –, × and ÷ can be held up in response to questions about the operation needed to solve a problem.

A large **100 square**, displayed where children can touch it, is essential for work in Years 2 to 4 on patterns such as 43 + 8, 43 + 18, 43 + 28, 43 + 38… or to illustrate addition or subtraction of two-digit numbers: for example, when 38 + 23 is treated as 38 + 10 + 10 + 3. Similarly, 72 – 47 can be treated by counting back first 4 tens then 7 ones to reach 25. Another way to illustrate 72 – 47 is to count up from 47 to 67 in tens, then from 67 to 72 in ones.

From resources kept in each classroom it ought to be possible to equip each group with **small apparatus** such as counters, interlocking cubes, wooden cubes, pegs and pegboard, straws, rulers, coins, dominoes, dice, and calculators when needed. Each class needs ready access to a variety of **squared paper**, and a good range of **number games, measuring equipment, sets of shapes and construction kits**.

Base-10 apparatus can be used to show the relative sizes of 1, 10, 100 and 1000, and partitioning of numbers: for example, how to break 374 into 300 + 70 + 4, or into 300 + 60 + 14. However, some children learn to manipulate the pieces without ever transferring their understanding to the number system. Always use digit cards alongside the pieces to help overcome this. The same applies to a **spike abacus**.

Your library corner might have some **interest books on mathematics** and **mathematical dictionaries** suitable for the age of the children. For activities and practice exercises for class work and homework there are many useful books of suggestions for teachers and pupils produced by educational publishers, mathematical associations, local education authorities and others. You may need particular equipment, books and materials for pupils with special needs.

Don't forget that the aim is for children to become less reliant on fingers and apparatus and to calculate mentally. Try to develop an approach in which mental methods are always considered first. Use strategies like: 'First think, and try to work it

out in your head. Now check on your number line.' Or: 'Close your eyes and imagine five counters on the table… a number line on the wall… two dice you have shaken… a bag of silver coins…'

Information and communications technology (ICT)

ICT includes the calculator (see page 8) and extends to the whole range of audio-visual aids, including audio tape, video film and educational broadcasts. You can use ICT in various ways to support your teaching and motivate children's learning. For example, by using a computer pupils in Key Stages 1 and 2 can:

- explore, describe and explain number patterns: for example, by watching a counting 'meter' with sequences of numbers shown slowly one at a time, or experimenting with patterns of multiples highlighted on different number grids;
- practise and consolidate their number skills: for example, by using software designed to 'teach' or practise a particular skill and give rapid assessment feedback to you and them;
- explore and explain patterns in data: for example, by accessing, displaying and interpreting ready-made sets of data, displaying quickly a bar chart or pictogram showing the outcome of a class vote, or using a sensor connected to a computer to measure, display and show trends in room temperature;
- estimate and compare measures of length or distance, angle, time, and so on: for example, by devising a sequence of instructions to move a floor robot or screen 'turtle' along a path, then modifying their instructions in the light of the robot's response;
- experiment with and discuss properties of patterns in shape and space: for example, by using software to transform shapes and create geometric patterns, or watching a film of a square being halved in different ways;
- develop their mathematical vocabulary, logical thinking and problem-solving skills: for example, by using a 'branching tree' computer program to sort shapes or numbers, or exploring a simple simulation to discover the mathematical relationship that underpins it.

An aim of the daily mathematics lesson is to keep the class working together and to link but limit to no more than three the number of different activities going on during group work. Most schools with pupils in Key Stages 1 and 2 don't have enough computers for all the children in a class to do the same activity simultaneously. But you or another adult can make good use of a single computer in the daily lesson by working with the whole class, if the monitor's screen is large enough. An alternative is for you to work with part of the class – perhaps a group of six to eight pupils. As with other ways of teaching mathematics, your role is to demonstrate, explain and question, stimulate discussion, invite predictions and interpretations of what is displayed and ask individual children to come to the keyboard to enter an instruction or a response.

A small group of pupils working together can also make effective use of one or two computers in the daily mathematics lesson, provided that the activity is consistent with the lesson's objectives, the activities of other groups and the overall number of activities. You need to intervene in the computer work from time to time to teach and develop the children's learning, and make sure that they are all participating. You should also invite them to contribute to the plenary part of the lesson.

Individual use of computer programs is usually inappropriate in the daily lesson, except where pupils with profound special educational needs or exceptional ability are doing individualised work. But programs which allow any pupil to practise number skills independently, or to investigate a mathematical problem with a partner, have a valuable part to play in breaks and after-school clubs, and at home.

You should use computer software in your daily mathematics lesson only if it is the most efficient and effective way to meet your lesson's objectives. For example, an aimless exploration of an 'adventure game', or repetitive practice of number bonds already mastered, is not good use of lesson time. And it is time-consuming for children to develop their understanding of addition and subtraction by taking turns to instruct a floor robot to move along a number track; a much quicker way of achieving the same mathematical objective is for as many children as possible to walk up and down the track, and to observe others doing so.

The supplements of examples that are part of the Framework include some references to ICT, where its use can be managed efficiently by a teacher working with a whole class or a large group, and the activity helps to deliver the National Curriculum for ICT. Specific programs are scarcely mentioned, since the focus in the supplements is on mathematical outcomes, not the resources that can be used to achieve them. But you could annotate the supplements of examples in suitable places with references to the software and other resources that you have in school.

ICT is, of course, more than a teaching tool. For many pupils with special educational needs it is an essential communication aid. The Internet offers teachers access to research articles and materials to download for classroom use, such as mathematical problems for children to solve with accompanying notes for teachers. The web-sites of the mathematical associations give useful information and can guide you to other useful sites. The development of the National Grid for Learning, which will include the Virtual Teacher's Centre, will also offer schools practical sources of advice and the opportunity to exchange ideas with others.

Assessment

Assessment, recording and reporting are important elements of teaching but they have to be manageable if the information they yield is to be useful to you, the pupils and others. Practice needs to be agreed across the school to ensure consistency and efficiency.

As with planning (see page 38), it is best to think of assessment at three connected levels: short-term, medium-term and long-term. Your assessments can then inform your teaching plans at each level, in a continuous cycle of planning, teaching and assessment.

Short-term assessments

Short-term assessments are an informal part of every lesson. Their purposes are to:

- check that children have grasped the main teaching points in a particular lesson or unit of work, whether they have any misunderstandings that you need to put right, and whether they are ready to move on to the next activity;
- check that children are remembering number facts and can use mental calculation strategies;
- give you information which will help you to adjust day-to-day lesson plans and brief any support staff or adult helpers about which children to assist, and how to assist them.

For these short-term assessments what you assess will be closely matched to your teaching objectives. There are two main ways to make them.

- **During every lesson** you will absorb and react to children's responses, see whether they are confident or hesitant with new work, decide whether they need extension work or more help, or if groups need to be adjusted, and so on. With four groups in a class you might, for example, aim to keep a special eye on one of them on a different day each week, perhaps questioning the children informally to check specific knowledge, skills and understanding. If you notice any difficulties or misunderstandings, try to adjust your lesson and address them straight away, if necessary continuing in the next lesson or two. Use the plenary part of the lesson to acknowledge individual and collective achievement and effort.
- **At intervals** you can supplement your daily observations. For example, with older children, an out-of-class or homework exercise or activity at the end of a unit of work can give you useful information on who has learned what and who needs extra support. Or you might occasionally give the class a short, informal test of rapid recall of number facts and mental calculation skills. In a mixed-age class, older pupils could answer written questions, while the younger ones write answers to questions which you give to them orally. Homework and informal tests of this kind should be followed immediately by marking and discussion with the whole class to give pupils feedback on their performance and what they need to do to improve. At the same time you can make sure that any errors are put right and the merits of different methods discussed.

Short-term assessments don't need to be recorded, since they are for your immediate action and attention. But you may decide to keep your own informal jottings when a child surprises you, perhaps with his or her knowledge, or with something that is unexpectedly difficult. These informal, personal recordings can help you to clarify patterns in performance over time or responses to specific teaching or support.

Medium-term assessments

The purposes of medium-term assessments are different. Their focus should be on what you are unsure about, not on what you already know. They are mainly to:

- review and record the **progress** children are making over time in relation to the key objectives, what they know and can do, whether they can apply their skills in a new context, and whether any weaknesses remain;
- identify children's progress against specific individual targets, including those in IEPs, so you can give them and their parents feedback and set new targets;
- help you to plan work over the next half term;
- provide you with information to feed into end-of-year assessments.

It is not necessary or even feasible to check and record each pupil's individual progress against every single teaching objective in mathematics. Most children should be living up to expectations for their age group; what they know, understand and can do in general is already recorded as a class record of progress when you evaluate your medium-term teaching plan (see page 42).

Your medium-term assessments should centre on the most important aspects of mathematics and help you to identify children's particular strengths and weaknesses. They should relate to the **key objectives** that you have included in the half term's work (for details of the key objectives, see the separate section). These objectives are central to all children's progress in relation to the National Curriculum level descriptions, and hence to their performance in tests and teacher assessments.

Medium-term assessments should be timed to influence planning. For example, on two days in each half term you could plan **group and individual assessment activities and written tasks**. These might involve several different ideas and skills linked to one or more of the key objectives. You should tell the class the particular focus of your assessment: for example, accuracy, working shown clearly for written calculations, clear and concise explanations of the methods used to solve problems… When you plan 'assess and review' days, try to choose activities and tasks that children can largely tackle independently so that you can concentrate on observing how they set about their work as well as its quality. For example, in Year 4, you could make use of the Assessment Units published by the QCA.

As you assess the work and review progress, bear in mind your planned expectations for what pupils should be achieving in relation to the key objectives. Refer to the QCA exemplification of children's work which accompanies this Framework, and to the Framework's supplements of examples. Aim to judge how well your pupils can use and apply what they know, understand and can do, and what difficulties remain. Try to rectify any difficulties as soon as possible and take account of them in your next phase of medium-term planning.

You will need to mark any written task that is part of medium-term assessment to give feedback to children on what they have achieved and how to improve. You will probably want children to make corrections to their work, so constructive written comments are more helpful than mere ticks and crosses, or scores 'out of 10'. The marking, feedback and corrections should be done as soon as possible while children can still remember how they approached the task.

Children's progress towards the key objectives needs to be recorded. Since there are relatively few key objectives for each year, records will not be too onerous to maintain. To update them every six weeks or so after your 'assess and review' days is sufficient. If you have kept any personal jottings on your short-term assessments you can use these to help you.

The easiest system to use is a **class record of the key objectives**: a summary sheet for the whole class, with the key objectives appropriate for the class down one side and children's names across the top. For example:

Key objectives: Year 4											
Use symbols correctly, including <, >, =											
Round any positive integer less than 1000 to the nearest 10 or 100											
Recognise simple fractions that are several parts of a whole, and mixed numbers; recognise the equivalence of simple fractions											
Use known number facts and place value to add or subtract mentally, including any pair of two-digit whole numbers											
Carry out column addition and subtraction of two integers less than 1000, and column addition of more than two such integers											
Know by heart facts for the 2, 3, 4, 5 and 10 multiplication tables											
Derive quickly division facts corresponding to the 2, 3, 4, 5 and 10 multiplication tables											
Find remainders after division											
Know and use the relationships between familiar units of length, mass and capacity											
Classify polygons, using criteria such as number of right angles, whether or not they are regular, symmetry properties											
Choose and use appropriate number operations and ways of calculating (mental, mental with jottings, pencil and paper) to solve problems											
Other											

Your class record can be dated when you feel confident that a child has achieved a key objective. This is a sufficient record of most children's progress but you will need some **supplementary notes** for the few individual pupils whose progress towards the key objectives differs markedly from the majority of the class. Your notes should give the reasons for the difference, perhaps referring to a sample of work that you decide to retain.

Your class record can be kept in a folder with your supplementary notes. You will need to pass your folder to the next teacher at the end of the year.

Individual targets for pupils

Setting individual targets for pupils is another way in which you can help children to achieve the key objectives over the medium term.

Try to have a discussion of 5 to 10 minutes with each pupil during the course of each term to set his or her personal targets. Although you may want to arrange your discussion with some pupils on an individual basis – for example, pupils with special needs whose IEPs need updating, or pupils who would benefit from a degree of privacy – for other pupils you can organise the discussion in small groups. For example, on one of your 'assess and review' days you could use group time to circulate and ask pupils to name two or three simple improvements to work on over the next term. You could also use registration time to talk to three or so pupils each week, either individually or as a small group.

The aim is to discuss each pupil's individual progress in meeting their targets and praise success, then set a couple of new or amended targets for the pupil to aim for. You could also offer children and their parents some practical advice on the steps they might take to achieve their targets.

The targets will usually be linked to those key objectives that you will focus your teaching on over the next few weeks. They may be very specific: for example, to learn by heart multiplication facts in the 3 and 4 times-tables, or to become accurate at subtracting a pair of four-digit numbers using a pencil and paper method. For some pupils a target may need to be broken down into stages: for example, to learn by heart the 3 times-table up to 5×3. For others, it may be appropriate to choose a target linked to the key objectives for the year group below or above. Whatever the targets, they need to be straightforward and not too many at one time so that everyone understands them. You can then readily monitor and discuss them with children during mathematics lessons.

You need to note the targets you set so that you can refer to them in subsequent discussions with the children or their parents. In most cases you could highlight boxes on your class record of key objectives to indicate which of them have been given to children as targets. Exceptionally, there may be some pupils whose personal targets need to be recorded in your supplementary notes or IEP.

Keeping parents informed and involved

Your class record of key objectives and your supplementary notes will help you to keep parents informed and involved in their child's progress. When opportunities occur, use the record and notes as a point of discussion at a meeting with parents, emphasising what their child has already achieved as well as what she or he needs to work towards next. Try to gain parents' active support for helping their children to achieve the targets. For example, you could give parents a copy of their child's targets, with some suggestions for how families can help. You could also include work related to the targets in the homework activities you give your class to do.

Long-term assessments

Towards the end of the school year you will need to assess and review pupils' progress and attainment against school and national targets, drawing on your class record of key objectives and supplementary notes.

Long-term assessments are important in each year group, not just at the end of Key Stage 1 and Key Stage 2. Their purposes are to:

- assess pupils' work against the key objectives for the year;
- at the end of a key stage, assess pupils' work against national standards;
- give you supplementary information about individual children's attainment and progress so that you can report to parents and the child's next teacher;
- help the school to set targets for the National Curriculum tests in future years;
- allow the headteacher to brief the governing body, the staff and others on overall progress and attainment in the school as a whole, including progress towards school, LEA and national targets.

The main ways in which these long-term assessments are made are through end-of-year tests for children from Year 2 onwards, and teacher assessments.

- The compulsory **National Curriculum mathematics tests** for pupils in Years 2 and 6 can be supplemented by the **optional tests for Years 3, 4 and 5** provided by QCA. The age-standardised scores which result from these tests will help you to monitor whether pupils individually and collectively are attaining at, below or above the 'national average' score of 100, and how their attainment compares with their attainment in the previous year. Results expressed as National Curriculum levels will help you to judge overall standards and progress towards school, LEA and national targets.

  Each year QCA publishes for each Key Stage a *Standards Report* analysing pupils' performance on the National Curriculum test questions. These reports can help you to identify particular weaknesses which you may need to tackle in your next phase of teaching.
- You will also need to make a **teacher assessment** to sum up your judgement of children's attainment. For Year 2 and Year 6 your end-of-year assessment will need to be made against the National Curriculum level descriptions. The cumulative picture which you carry in your head of the progress of each child in your class can be extended and secured by looking through samples of children's work. You will then need to update and complete your class record of key objectives, and any supplementary notes you have made on individual pupils.

Before you make your end-of-key-stage teacher assessment, it is helpful if all staff across a key stage can examine together a sample of pupils' work from each class. This moderation exercise helps to make sure that judgements against the National Curriculum level descriptions are consistent through the school. Exemplifications of children's work published by QCA are particularly helpful at this time.

Passing on information about pupils' attainment and progress

You will be able to base your end-of-year reports to parents on your long-term assessments, making sure that they are free of jargon so that parents understand them easily.

For the next teacher, your class record of key objectives ought to give a good indication of how children have responded during the year and any difficulties that remain. Passing on a recent example of each child's written work to show, in particular, the stage reached in recording calculations is also helpful.

Planning

How the Framework is set out

The Framework consists of a set of **yearly teaching programmes** or 'programmes of study' summarising teaching objectives for each year from Reception to Year 6. From Year 1 each double-page programme covers the full range of the National Curriculum for mathematics that is relevant to the year group. The **key objectives** to which you should give priority are set out in a separate section and are also highlighted in bold in the yearly teaching programmes.

With each year's programme are two **planning grids** which, if you wish, you can use to help plan a term's lessons: a common one for use in the autumn and summer terms and one for the spring term (for Reception there is one for each term). Each grid indicates the topics to be taught in units of work, and the **recommended** number of days of lessons for each unit, except in Reception, where the length of units of work needs to be determined once children have settled in to school. The first and last units in each term are always shorter – just two or three days – to allow for the start and end of term. Two days are set aside in each half term for you to assess and review progress.

The units may be taught in any order but if you increase the number of days allocated to a particular unit you will need to decrease the number of days allocated to another. Overall, the units of work for each of Years 1 to 6 require 175 days of the school year, leaving about one week in each term for extra reinforcement or revision, making cross-curricular links or more extended problem solving. The grids for Reception allow time for settling in and for new children to join the class.

The grids for Years 1 and 2, for Years 3 and 4, and for Years 5 and 6 correspond very closely. This is to help teachers of mixed-age classes.

After the teaching programmes and grids come **supplements of examples**, for Reception, Years 1–3 and Years 4–6. The examples illustrate outcomes – a selection of what pupils should know and be able to do by the end of the year. For Reception, the examples illustrate Early Learning Goals for a child who becomes 5 in the autumn term, and who spends a full year in the class.

In each set of examples, the broad topics of counting, properties of numbers, place value and ordering, and so on, are printed in the top right corner of the relevant pages. Teaching objectives for the topic are listed in the left-hand column. Examples to illustrate each objective are shown in the columns alongside, set out so that you can recognise progression from one year to the next.

The examples are a selection, not a full set, and are not intended to be taught as a 'scheme of work' or used on a series of worksheets. Their main purpose is to help you first to interpret the level of the work and then to plan, pace and assess it so that there is steady progression throughout the school. The yearly teaching programmes and the termly planning grids both include cross-references to relevant pages in the example supplements.

The Framework's five strands

The Framework has five strands. The first three have direct links to the National Curriculum programme of study for number. The fourth strand is linked to measures, shape and space, while the fifth incorporates handling data. Using and applying mathematics is integrated throughout. The strands, and the topics they cover, are:

Numbers and the number system

- counting
- properties of numbers and number sequences, including negative numbers
- place value and ordering, including reading and writing numbers
- estimating and rounding
- fractions, decimals and percentages, and their equivalence; ratio and proportion

Calculations

- understanding number operations and relationships
- rapid mental recall of number facts
- mental calculation, including strategies for deriving new facts from known facts
- pencil and paper methods
- using a calculator
- checking that results of calculations are reasonable

Solving problems

- making decisions: deciding which operation and method of calculation to use (mental, mental with jottings, pencil and paper, calculator…)
- reasoning about numbers or shapes and making general statements about them
- solving problems involving numbers in context: 'real life', money, measures

Measures, shape and space

- measures, including choosing units and reading scales
- properties of 2-D and 3-D shapes, position, direction and movement

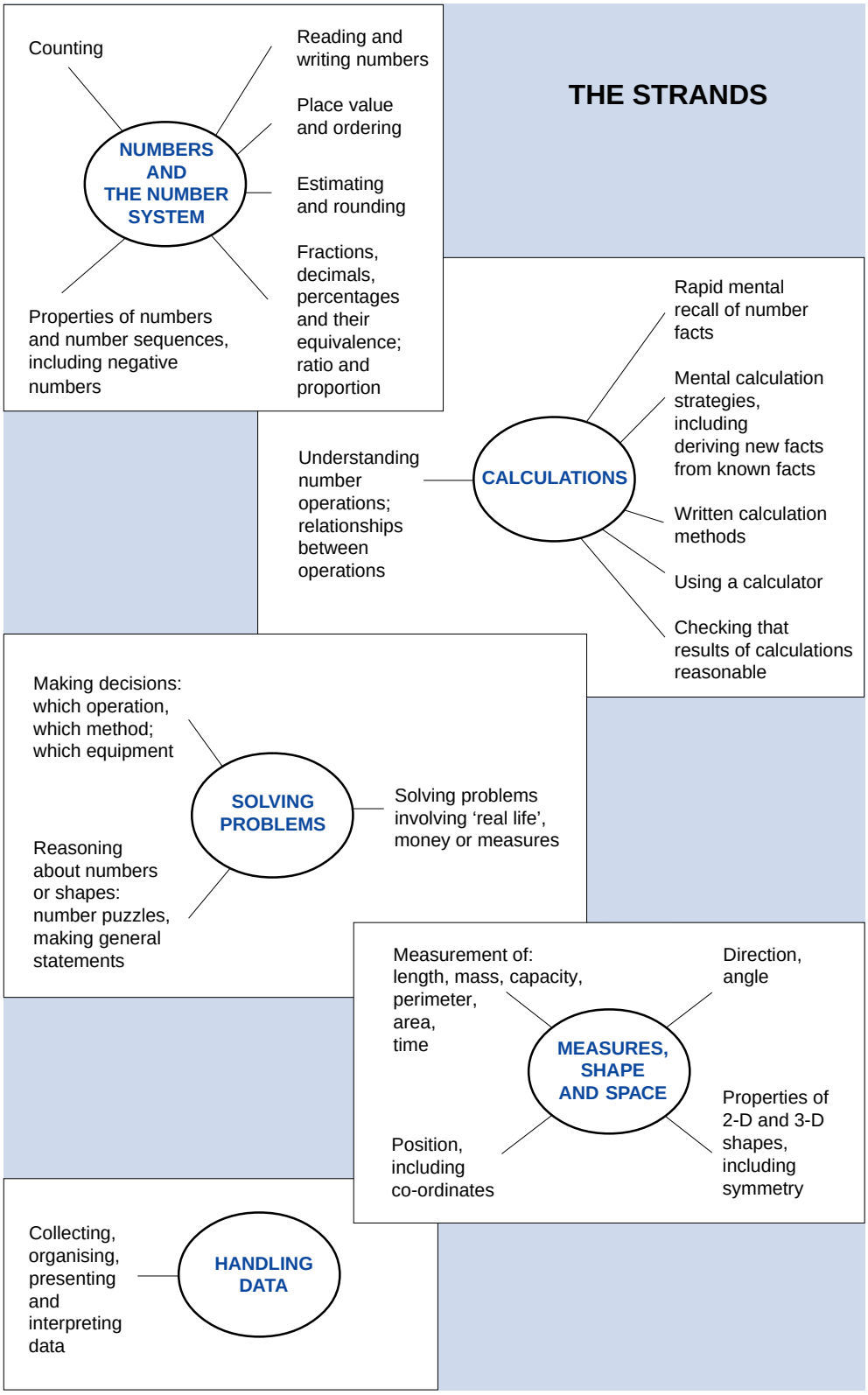
Handling data

- collecting, presenting and interpreting numerical data

Although the strands are described separately, mathematics has many connections within and across topics. For example, when pupils are being taught to multiply by multiples of 10, they will make connections within a topic by drawing on their knowledge of multiplication table facts and understanding of place value. Using counters to form rectangles to introduce factors and division of numbers helps to link different topics such as properties of shapes, numbers and calculation. The statement 3 + 2 = 5 represents and summarises a range of situations which appear different but which are equivalent, such as making three whole turns followed by two whole turns, or starting with £3 and being given £2 more. Showing pupils how to multiply using partitioning, so that 12×3 becomes $(10 + 2) \times 3$, prepares the way for later connections such as long multiplication or work in algebra.

You need to be clear about what can be connected within and across topics, to make these connections visible for pupils and to help them to make some of their own. Providing different examples and activities and expecting pupils to make the links is not enough; pupils need to be shown them and reminded about their work in earlier lessons. Explanations, demonstrations and illustrations of connections should all be part of the direct teaching pupils receive during the main teaching activity.

The diagram below illustrates the five strands. **Using and applying mathematics is integrated throughout**: for example, in making and justifying decisions about which method, equipment or unit of measurement to use; in describing properties of numbers or shapes and in reasoning about them; in explaining methods of calculation; in devising and refining methods of recording calculations; in checking results…



Principles of good planning

To put the Framework into practice you need three connected levels of planning:

- the Framework: what you should teach long term;
- medium-term plans: termly outlines of units of work and their main teaching objectives, and when you will teach them;
- short-term plans: weekly or fortnightly notes on tasks, activities, exercises, key questions and teaching points for 5 to 10 lessons, including how pupils will be grouped, which of them you will work with, and how you will use any support.

Your medium-term plan is the basis for your more detailed short-term plans. It identifies **what** you will teach across the term and when it should be happening. Your short-term plans can focus on **how** you will teach – in particular, what you will do and what the children will do.

Whether or not you choose to use the termly planning grids your school's planning procedures for mathematics should meet these criteria. There should be:

- common formats for planning a balanced programme of objectives for each term, and common formats for planning one or two weeks of lessons;
- arrangements to support planning: for example, through planning in teams with the help of the co-ordinator, SENCO or deputy;
- agreed procedures and deadlines for producing plans;
- monitoring arrangements to evaluate planning and progression throughout the school, and the impact of plans as they are put into practice in classrooms.

When you are planning lessons you should give some thought to what pupils have already been taught so that you can build on the concepts, knowledge and skills they have already acquired. You will need to keep these questions in mind.

- What mathematics have these pupils been taught before?
- How will my lessons build on what they already know, understand and can do?
- How can I use previous lessons to help pupils establish links between topics: for example, can I use similar examples, common vocabulary and earlier examples to illustrate and demonstrate connections?
- Can I use opportunities in other subjects to introduce or reinforce mathematical ideas? (See 'Making links between mathematics and other subjects', page 16.)

Developing medium-term plans and a scheme of work

If you choose to use the termly planning grids, copy the relevant grid before the start of term, enlarging it to A3 size. Identify the particular objectives that will be the main focus in each unit of work, choosing from the objectives in the relevant yearly teaching programme. Every objective should be included and covered at least once by the end of the year. For Years 1 to 6, choose and then record objectives for oral and mental mathematics sessions in the box for each half term.

The purpose of the units is to make sure that the balance and distribution of work across each term is appropriate, but they may be taught in any order. Since you will need to identify time in other subjects to supplement mathematics, particularly for

practical work using measures and properties of shapes, this may have implications for the order of the units.

The page references on the termly planning grids should help you to refer to the examples in the supplement when you plan day-to-day lessons in detail.

Evaluating your medium-term plan

At the end of each unit of work you should evaluate it, based on your short-term assessments of the pupils. For example, you could highlight or code your termly plan to show whether in general:

- pupils responded well and met the objective in full;
- pupils were responsive but the objective still needs more attention;
- an objective was not covered, or pupils did not meet it.

You can then see at a glance what pupils in general can do and what still needs more work. You should develop next term's plan in the light of these evaluations, taking account of your assessments of children's progress towards the key objectives.

Creating a scheme of work

By keeping the termly and weekly plans for the whole school in a folder and replacing them with modified updates when you develop them next time round, you can create a developmental scheme of work for mathematics for your school that is never out of date.

Relationship to National Curriculum level descriptions

The overall target is for at least 75% of 11-year-olds, by 2002, to achieve level 4 in the National Curriculum tests for mathematics. The year-by-year programmes are designed with this target in mind.

The programmes also take account of the need in mathematics to revisit topics regularly to revise and consolidate skills and then extend them. They may, at first, seem ambitious. But if 11-year-olds are to achieve a secure level 4 when tested in the summer of Year 6, the teaching programme needs to be pitched at a level which is a little beyond this.

The expectations in the yearly teaching programmes correspond to these levels.

- **Year 1** level 1, and start on level 2
- **Year 2** consolidation of level 2, and start on level 3
- **Year 3** revision of level 2, but mainly level 3
- **Year 4** consolidation of level 3, and start on level 4
- **Year 5** revision of level 3, but mainly level 4
- **Year 6** consolidation of level 4, and start on level 5

Where to begin

The Framework is a guide to what to teach each class. However, there are schools where at present relatively few Year 6 pupils attain level 4 in the National Curriculum tests. These schools will need to look carefully at the teaching programmes for Years 5 and 6 and judge the extent to which they are appropriate.

For example, in the first year of using the Framework it may be more appropriate for a Year 5 class to work on the Year 4 programme for a couple of terms, before moving on to the Year 5 programme. This decision would need to be reviewed at the start of the next school year when a new group of pupils who have had some teaching based on the Framework enters the Year 5 class.

The Government has given schools greater flexibility to plan their curriculum provision in ways that give greater emphasis to literacy and numeracy. Where necessary, schools should use this flexibility to bring pupils up to the appropriate standard as soon as possible.

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