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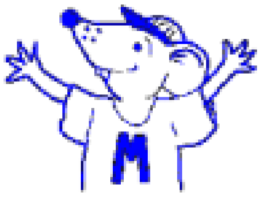
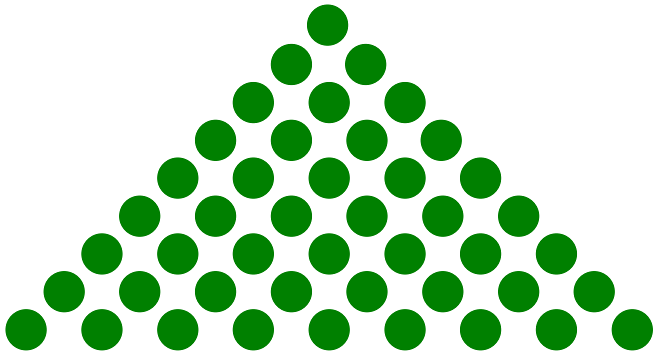
INVESTIGATION



Triangle Numbers



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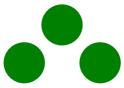
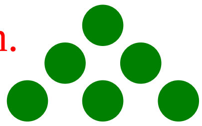
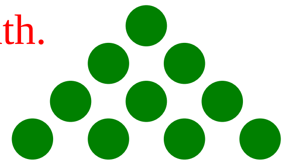
What is a Triangle Number?

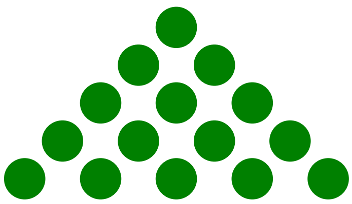
Begin with one dot at the top of a triangle. So far **1** dot altogether.

Put two dots underneath.  So far **3** dots altogether.

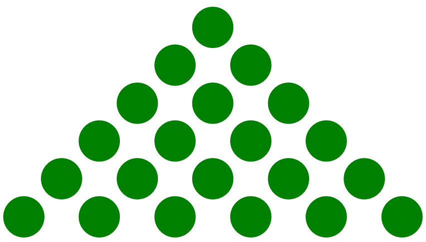
Put three dots underneath.  So far **6** dots altogether.

Put four dots underneath.  So far **10** dots altogether.

Keep putting one more dot in each row:



15 dots altogether.



21 dots altogether.

1, 3, 6, 10, 15, 21 are the triangle numbers.

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Here are some problems about Triangle Numbers.

1. Can you calculate the next six triangle numbers <u>without</u> a calculator?

2. Can you calculate the next twenty triangle numbers <u>with</u> a calculator?

3. Which is the first triangle number after 100 ?

4. Which is the first triangle number after 1 000 ?

5. Which is the first triangle number after 10 000?

6. Which triangle numbers are also square numbers?

7. Can you find any triangle numbers that are also prime numbers?

Triangle numbers are the whole numbers added together.

You should be able to see from the triangle pattern that triangle numbers are the whole numbers added together.

For example,

$$1 = 1$$
$$3 = 1 + 2$$
$$6 = 1 + 2 + 3$$
$$10 = 1 + 2 + 3 + 4$$
$$15 = 1 + 2 + 3 + 4 + 5$$
$$21 = 1 + 2 + 3 + 4 + 5 + 6$$

It would be a good idea to make sure you understand all this before going on.

Triangle numbers appear in all sorts of situations, not just in triangles!

On the next few pages are some investigations for you to try.
Try each starter and see what you can discover.

Remember to work carefully or you may miss some important point.

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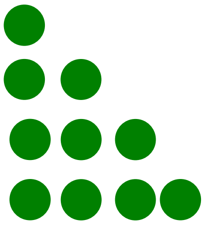
Triangle numbers and square numbers.

Sometimes it is more convenient to draw a triangle in which the left hand edge is vertical, like this:



Write down the triangle numbers in a sequence:

1, 3, 6, 10, 15, 21, ...

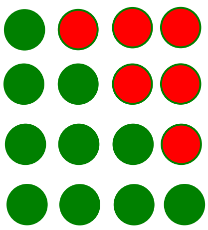
Draw a rectangle around two adjacent triangle numbers:

1, 3, $\boxed{6,\ 10,}$ 15, 21, ...

Add up the two numbers in the rectangle.
Try the same idea for other pairs, such as 10 + 15.
What do you notice?

Can you explain what you have discovered?

This diagram may help you:



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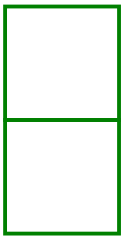
Triangle numbers and Dominoes.

You are probably familiar with the game of dominoes, but did you ever wonder how many dominoes there are in a set.

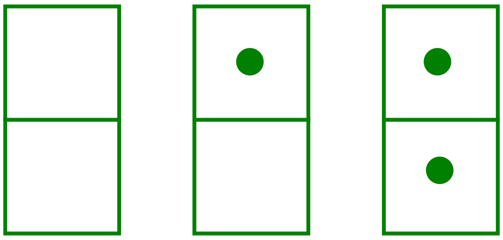
There are two popular versions of dominoes - a set in which the highest number of dots is six and a set in which the highest number of dots is nine.

But we are mathematicians, so we can have dominoes with any number of dots we like.

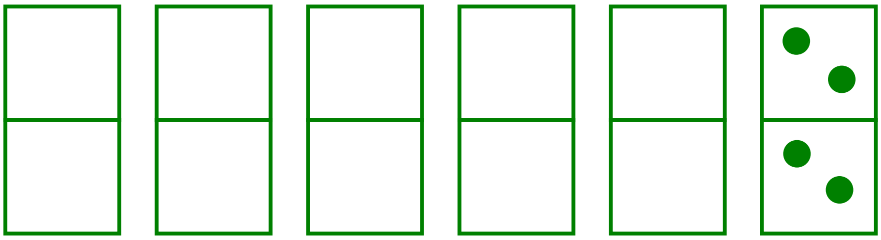
Let's make up a set of dominoes with only blank squares:



Now let's make a set with blanks and ones.



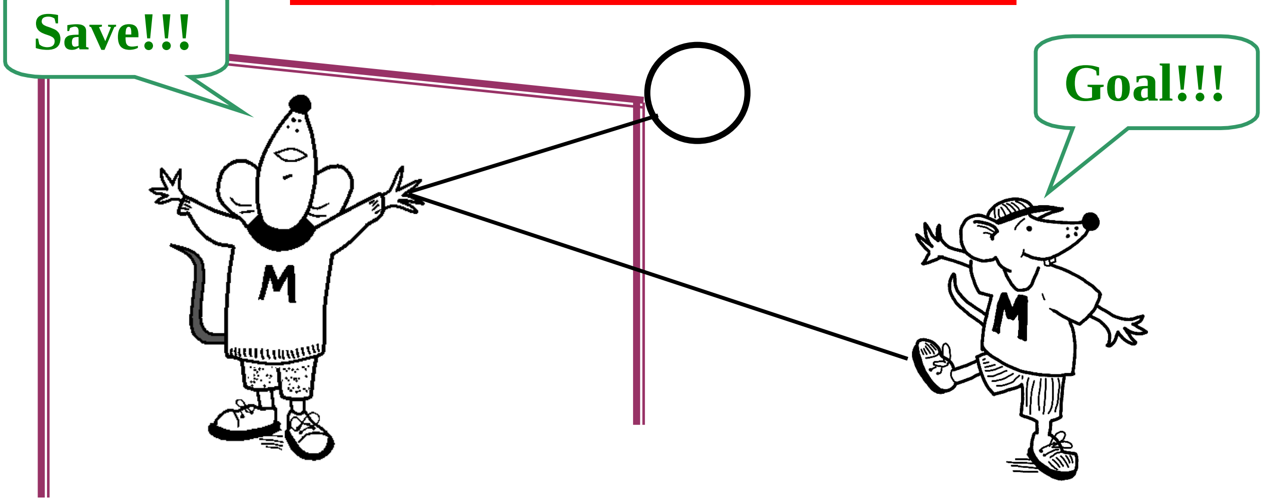
Can you complete this set with blanks, ones and twos?



Now you can work out how many dominoes there will be in a set with up to three dots, four dots etc.

How many dominoes are there in the common six-dot and nine-dot sets?
Has this anything to do with triangle numbers?

Triangle numbers and Football.



If the highest score by either of the teams in a football match was 2, what could the half time scores have been?

Hopefully, you will agree the half time scores could have been:

0 - 0, 1 - 0, 1 - 1, 2 - 0, 2 - 1, 2 - 2

(Notice that in this investigation we count 2 -1 and 1 - 2 as the same score!)

Can you work out the half time scores when the highest score by either team is 1 goal, 2 goals, 3 goals, 4 goals etc.

Put your results in a table like this:

Max	Possible Half Time Scores																				
0																					
1																					
2																					
3																					
4																					

Can you see the connection with triangle numbers?

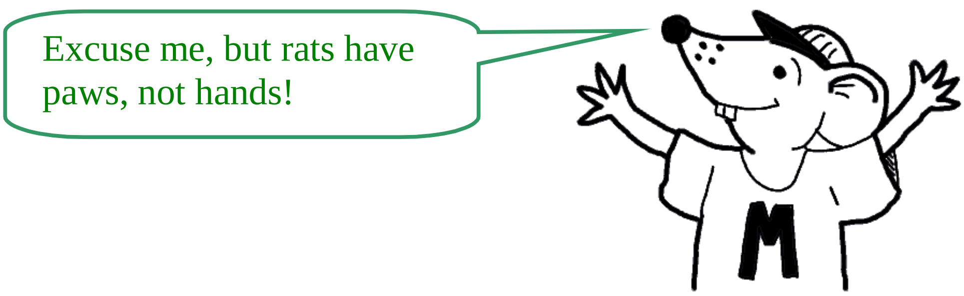
Triangle numbers and Handshakes.

The Maths Rats are having a party. They all shake hands as they meet each other.

If there are just two Maths Rats, how many handshakes will there be?

If there are three Maths Rats, how many handshakes will there be?

If there are three Maths Rats, how many handshakes will there be?



Can you see a connection with triangle numbers?

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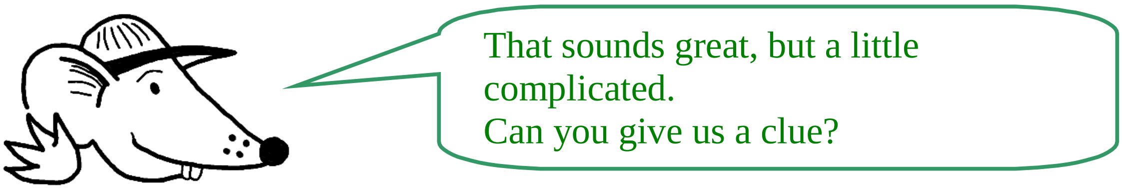
How to calculate Triangle Numbers.

Wouldn't it be great if you could work out what the fiftieth triangle number is without having to work out the forty nine before it?

Or if you could work out the hundredth, or thousandth without having to work out all of the others before them?

For this we need a formula.



We need to spot something about the way triangle numbers are made up.

Let's write down the first few:

1, 3, 6, 10, 15, 21, 28, 36, ...

Here's the clue: Double these numbers:

2, 6, 12, 20, 30, 42, 56, 72, ...

Now you should be able to see that each number is made from two numbers multiplied together:

(1×2), (2×3), (3×4), (4×5), (5×6), (6×7), (7×8), (8×9), ...

You should also be able to see that if we want the fifth triangle number, we simply multiply 5 by 6 (one more). This gives us twice a triangle number, so we need to have the answer.

$5 \times 6 \div 2 = 15$

Use this method to work out the seventh triangle number.
See if you get 28.
Can you calculate much bigger triangle numbers like the 100th or 350th ?

Triangle Numbers and Gauss.

Gauss was a great mathematician. He discovered many new things.

It is said that when he was at school, his teacher made him add up all the numbers from 1 to 100 as a punishment. (Even great mathematicians did not always have an easy time at school!).

His teacher was very surprised when the young Gauss gave the correct answer in a very short time.

How did he solve the problem?

It is said he wrote down the numbers from 1 to 100 and underneath put the numbers from 100 to 1, like this:

1,	2,	3,	4,	5,	6,	....	96,	97,	98,	99,	100
100,	99,	98,	97,	96,	95,	....	5,	4,	3,	2,	1

Then he added up each pair vertically and saw that they all came to 101.

1,	2,	3,	4,	5,	6,	....	96,	97,	98,	99,	100
100,	99,	98,	97,	96,	95,	....	5,	4,	3,	2,	1
101,	101,	101,	101,	101,	101,		101,	101,	101,	101,	101

He saw that there were 100 sets of 101, which is 10 100.
Because he had used the sequence twice, he had to halve the answer.

10 100 ÷ 2 = 5 050 His teacher was surprised!

Gauss had, of course, added up all the numbers from 1 to 100 and found the 100th triangle number!!!!!

Try this method yourself to find the tenth triangle number.

Then use it to find larger triangle numbers.

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Triangle Numbers and Perfect Numbers.

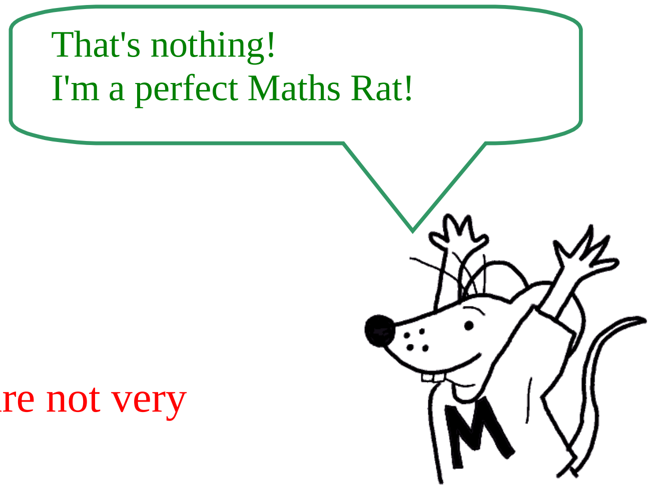
A Perfect Number is a number whose factors add up to the number itself.

(When you are working out the factors you should include 1, but not the number you started with.)

Here is an example:

The factors of 6 are 1, 2 and 3. 1 + 2 + 3 = 6, which is the number we started with.

So 6 is a perfect number.



Unfortunately, perfect numbers are not very common.

The next one after 6 is between 25 and 36. Can you find it?

The one after that is approximately 500. Perhaps you and your friends could each test a number around 500 to see if you can find it?

The fourth perfect number is 8 128. Can you check it?

The fifth is 33 550 336

Pretty big, huh!

What can you notice about the perfect numbers?

Are they triangular numbers?
Are all triangular numbers also perfect numbers?

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Answer Guide

Here are some possible answers and notes for guidance. Children may come up with many new ideas - this is, after all, the idea of an investigation.

Page 1:

1. 28, 36, 45, 55, 66, 78

2. 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 496, 528

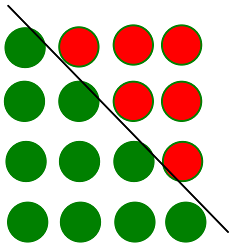
3. 105 **4.** 1035 **5.** 10 011 (Very difficult, but a good exercise in co-ordinating a group of children to keep together when carrying out a repeated calculation of this type!)

6. 1, 16, 1 225 (There aren't many in the lower range of numbers. The secret is to find a number which is the product of two consecutive numbers, one a square number and the other a number that is twice a square number. Multiply them and then halve the answer.
For example, $36 = 8 \times 9 \div 2$. 9 is a square number and 8 is twice a square number. 8 and 9 are consecutive. Only older, very able pupils will be able to work at this level.)

7. Since triangle numbers are calculated from the product of two other numbers, it follows that no triangle numbers can be prime numbers.

Page 2:

Adding adjacent triangle numbers always gives a square number. This can be seen from the diagram by drawing a diagonal line separating the two triangle numbers:



Try this for other combinations such as 10 + 15 or 15 + 21.

Answer Guide (Contd)

Page 3:

Yes, the number of dominoes in a set is always a triangle number. A surprising result, perhaps?

Page 4:

This is really the dominoes problem again in disguise. Don't forget, in this investigation, it does not matter which team scores the goals, so 3 - 2 is the same as 2 - 3.

Page 5:

In this investigation, it is important to realise that if person A shakes hands with person B, person B also shakes hand with person A, but there is only one handshake.

Ask the children to try shaking hands with each other in small groups so they can see what is going on before formalising this on paper. Eg. if there are 6 people in a room, six people shake hands with five others, giving 30 handshakes altogether. But, as A with B is the same as B with A, we must halve 30 to get 15 and we have found a triangle number!

Page 6:

This page is an introduction to the algebra involved in calculating sequences, but this may be beyond some children at this stage. The formula that children will come to eventually is:

$$\frac{n(n + 1)}{2}$$

Page 7:

A neat method for calculating triangle numbers. Start with small ones such as:

$$\begin{array}{l} 1 + 2 + 3 + 4 + 5 \\ \underline{5} + \underline{4} + \underline{3} + \underline{2} + \underline{1} \\ 6 \quad 6 \quad 6 \quad 6 \quad 6 \qquad\qquad 6 \times 5 = 30 \qquad 30 \div 2 = 15 \end{array}$$

Page 8:

This page is quite tricky once one is past 28, the next perfect number after 6. Some children love a challenge and so this is included, but finding 496 can be difficult as one mistake invalidates all the calculations.