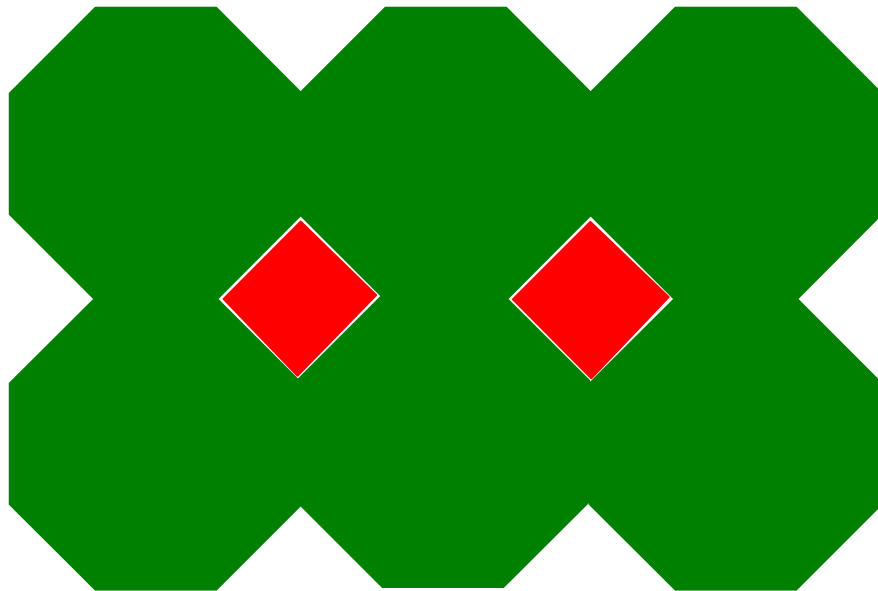




# INVESTIGATION



## Tessellations

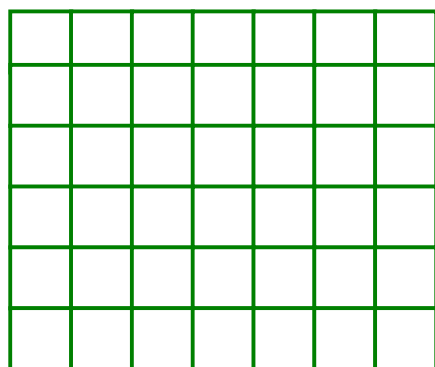


# MathSphere

## Tessellations

Tessellations are ways of arranging shapes so that there are no gaps between the shapes and they would cover a very large surface if you had the time to do this.

For example, the easiest tessellation of all is that of squares. If we take lots of squares that are all the same size, it is easy to fit them together to cover a large surface with no gaps, like this:

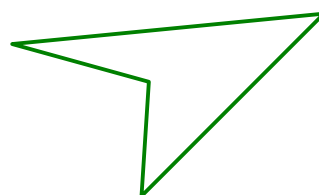
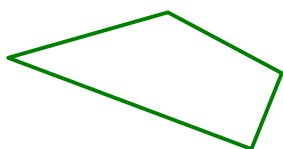


You can see this everyday in tiled bathrooms and kitchens and on black and white squared floors.

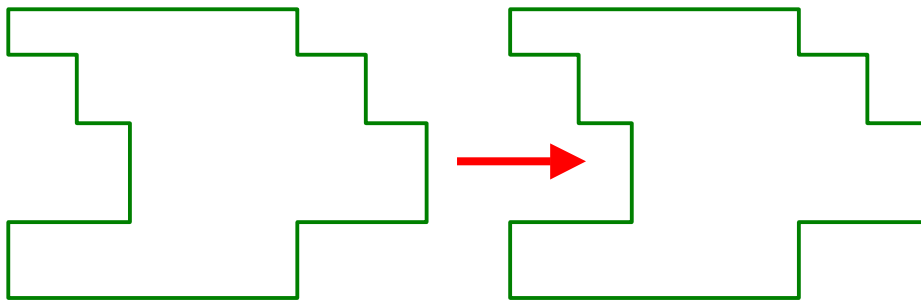
Many other shapes will tessellate.

### Here are some ideas to try:

1. Try different shaped triangles. Do all triangles tessellate? (Remember to keep to one shape and size of triangle for each tessellation - do not mix them!). Look at the angles of the shapes where the triangles meet - what do you notice?
2. Try different quadrilaterals. We know that squares tessellate and it should be easy to see that rectangles will too, but what about quadrilateral like these? :

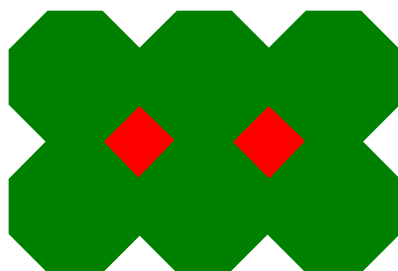


3. What about other polygons? Do all regular polygons tessellate?  
Try regular pentagons, regular hexagons, regular octagons etc.
4. What about irregular polygons? If a regular polygon will not tessellate, will an irregular one with the same number of sides? Can you find any?
5. You can make up your own interesting shapes that will tessellate. Begin with a simple shape that will tessellate such as a square. Cut a piece out of one of the sides and add it to the opposite side. You could now fit two or more of these shapes together in a line, like this:



Try also cutting pieces from the top and bottom and using curved shapes. Using this technique, you can make up some fabulous patterns!!!!

6. You could also try making up semi-regular tessellations. To do this you use two or more shapes such as regular octagons and squares. You often see this particular one in bathroom tiles.



## **Answer Guide**

As with all investigations, it is essential that children stick with one idea until they have investigated it to your satisfaction. With so many ideas to work on, it is easy for the children to flit from idea to idea and not learn much on the way. You may want to keep the second ideas sheet hidden until they have given the ideas on the first page a good working.

At each stage, ask the children to look at the angles at the points where shapes meet. There is a lot to learn here about interior/exterior angles and the sum of the angles at the meeting points being  $360^0$ .

1. All triangles do tessellate. In fact this is simply done by making rows in which each alternate triangle is placed upside down. Then put the rows together.
2. A surprising result here is that all quadrilaterals will tessellate! As the angles of a quadrilateral add up to  $360^0$ , you obviously need one of each angle at the meeting point. It is still not obvious, however, that this will mean they always tessellate, but they do!
- 3/4. The only regular polygons that will tessellate are equilateral triangles, squares and regular hexagons. However, many other polygons will tessellate. Get the children to find some. In question 5, for example, we have sixteen sided polygons tessellating.
5. By using this technique and cutting out curved shapes as well as those made from straight lines, it is possible to produce the most beautiful patterns.

Lastly, don't forget to show the children some of the work of Escher. You can explore the internet for suitable sites if you do not have any books of his works.