Back to Previous Menu

INVESTIGATION

Factors and Primes

I'm a perfect number.
My proper factors are
1, 2, 4, 7 and 14
They add up to me!

28

17

I'm a prime number.
I only have two
factors: 1 and me!

MathSphere

Factors and Primes

First some definitions!!!!

A factor is a whole number that will divide exactly into another whole number.
Eg. 3, 5 and 6 are some of the factors of 30.

The proper factors of a number are the factors of that number, not including the number itself.

Eg. The factors of 20 are 1, 2, 4, 5, 10 and 20.
The proper factors of 20 are 1, 2, 4, 5 and 10.

A prime number is a whole number bigger than 1 that only has two factors, 1 and itself.
Eg 11 is a prime number because its only factors are 1 and 11.

The first six prime numbers are 2, 3, 5, 7, 11 and 13

A composite number is a whole number that is not a prime number.
Eg. 4, 8, 9, 100 are all composite numbers.

The multiples of a number are the numbers in the times table of the number.
Eg. The multiples of 6 are 6, 12, 18, 24, 30, 36, ...

Make sure you understand these before you begin the investigation.

It would also be very helpful to work out all the prime numbers up to 100 (your teacher/parent has a complete list so you can check your answers).

The Problem

Find out everything you can about factors and prime numbers.

Good Advice:

Work in a logical way.

Try some ideas of your own.

Discuss what you have done with your friends.

Enjoy your work and record your results properly.

Some Ideas

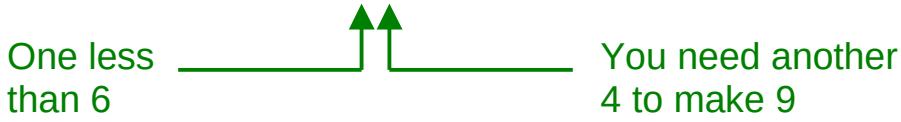
1. Work out how many factors the numbers between 1 and 50 have. Put your results in a table like this:

Number	Factors	Number of factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

Some Ideas (Contd)

2. Which numbers have just two factors?
   What can you say about them?
3. Which numbers have three factors?
   What can you say about them?
4. Which numbers have an odd number of factors?
   What can you say about them?
5. Which numbers have just one factor?
6. Make up a 'Bingo' game using factors.
7. A quick way to work out the multiples of nine (nine times table) is to put one less than the number you are multiplying by in the tens column and in the units column write what you need to make this up to nine.

   Eg. What are 6 nines? $6 \times 9 = 54$

   How far does this rule work?
   Can you make up rules for other tables?

8. Goldbach's Conjecture

   In 1742, Mr Christian Goldbach wrote to his friend Mr Euler that every even number can be written as the total of two prime numbers. In this problem we can include 1 as a prime number, although technically it is not prime.

   Test this for even numbers as far as you can.
   Eg. $6 = 3 + 3$, $8 = 3 + 5$, $18 = 13 + 5$

   Some numbers can be written like this in more than way:
   Eg. $20 = 19 + 1$, $20 = 17 + 3$, $20 = 13 + 7$.
   Can you find others?

Some Ideas (Contd)

9. Can you find any composite numbers with interesting patterns of factors?
   Eg $16 = 2 \times 2 \times 2 \times 2$
   Eg $24 = 1 \times 2 \times 3 \times 4$

10. Write down some prime numbers bigger than 4.
    Compare these with the numbers in the six times table.
    What do you notice?

11. If we write down the proper factors of 28 and add them up, the answer is 28 !!!!!
    The proper factors of 28 are 1, 2, 4, 7 and 14
    $1 + 2 + 4 + 7 + 14 = 28$

    Numbers that do this are called 'Perfect Numbers'.
    Unfortunately, there are not many of them.

    Can you find one which is less than 28 ?
    Can you find one which is quite near to 500 ?

    The next two perfect numbers are 8 128 and 33 550 336.

    What do you notice about them all?

    If you like a challenge, you might like to prove that 8 128 is perfect!

12. Problem for geniuses - no clues!
    There are certain pairs of numbers (let's call them A and B).
    The proper factors of A add up to B and the proper factors of B add up to A. These are called Amicable Numbers or Friendly Numbers.
    Can you find any? The first two are in the hundreds.

13. If you want to amaze your friends (or your teacher!) learn this number:

    170 141 183 460 469 231 731 687 303 715 884 105 727

    It is the largest prime number that was known before computers started to work them out.

Answer Guide

Here are some possible answers and notes for guidance.

Most children will be happy answering the first few questions; the rest are included for more able pupils who like to play with numbers and for those who like a challenge.

Here is a list of all the prime numbers up to 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

2. The numbers that have just two factors are prime numbers.

3. The numbers which have three factors are square numbers.

4. This is an extension of the last question as all numbers with an odd number of factors are square numbers.

5. Only 1 has one factor.

6. The Bingo game could be made from small cards giving multiplication sums and larger card giving a range of possible answers with which to play Bingo. Accept other sensible versions. If possible, allow children to make up their own cards as a lot of mathematics can be learned by thinking about which combinations make the best questions (eg. prime numbers can only be obtained by multiplying the number by one, whereas 24, for example, can be made from 1×24, 2×12, 3×8 or 4×6).

7. Interesting to see what the children come up with here.
For example, in the 5 times table, the tens digit is obtained by halving the multiplier (rounding down any halves) and putting a 0 in the units column if the multiplier is even and a 5 if it is odd.

8. Goldbach's Conjecture is still just a conjecture, ie. it has never been proven for all even numbers, although computers have tested it to extremely high numbers without ever finding an even number that cannot be written in this way.

Answer Guide (Contd)

9. A good chance here for some interesting calculator work.
For instance, begin with a sequence such as:
$1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$

Drop off the leftmost multiplier and add another onto the right end:

$2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5\,040$

$3 \times 4 \times 5 \times 6 \times 7 \times 8 = 20\,160$ etc

10. All prime numbers greater than 4 are one more or one less than a number in the six times table. The converse is not true!

11. The missing perfect numbers are 6 and 496.
All known perfect numbers are even and they are all triangle numbers.

12 Good luck. Why not try this as a class exercise in which different children calculate the proper factors of numbers, the results are tabulated and the children search for matches? The first two are 220 and 284, the second are 1184 and 1210.

13. This number is $2^{127} - 1$. Another way to look at it is this: imagine putting one grain of rice on the first square of a chessboard, two grains on the second square, four on the third and so on, doubling each time. Continue onto a second chessboard. The number above is one less than the number of grains of rice that would be on the last square.

Keen children could calculate how much this would weigh and cost if bought in a local supermarket. How does this compare with the total rice production for the world?