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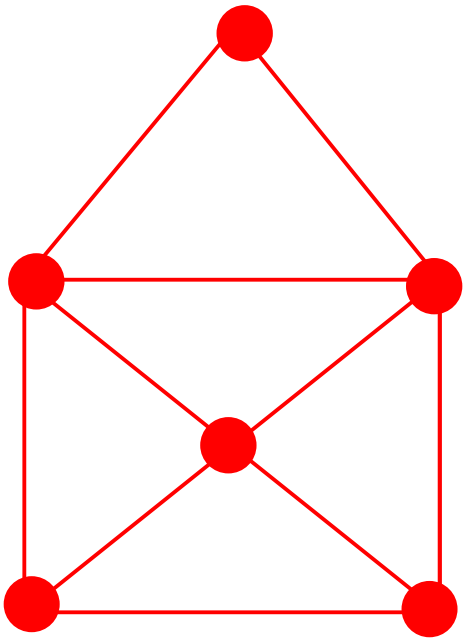
INVESTIGATION

Networks and all that stuff!



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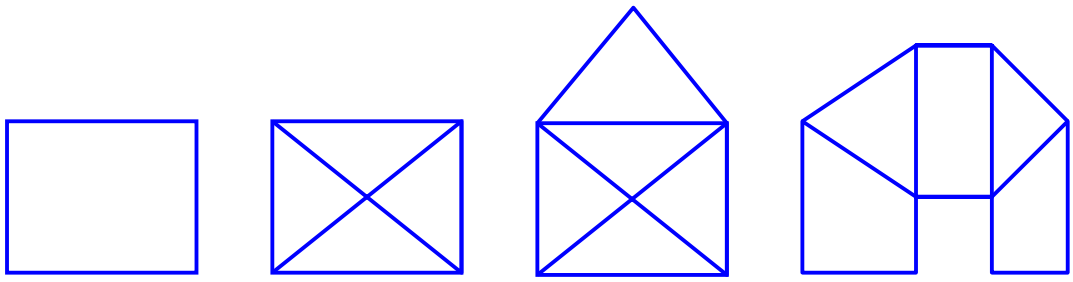
Networks and All That Stuff!

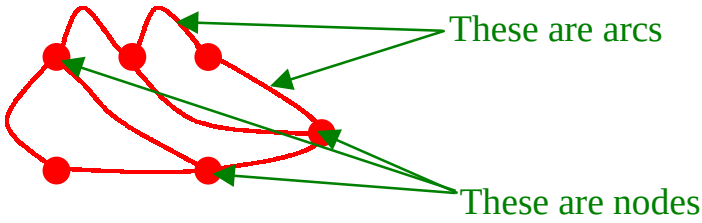
Which of these shapes can you draw without taking you pencil off the paper, without going over the same line twice and without cheating (such as folding the paper) ?



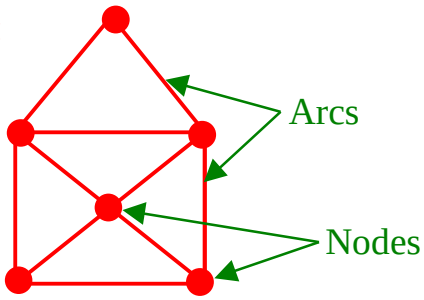
These shapes are called **networks**.

The third shape can be drawn without taking your pencil off the paper and without going over the same line twice.
Does it matter where you start?

Here is a network:



Here is another:



This is, of course the same as one at the top of this page..

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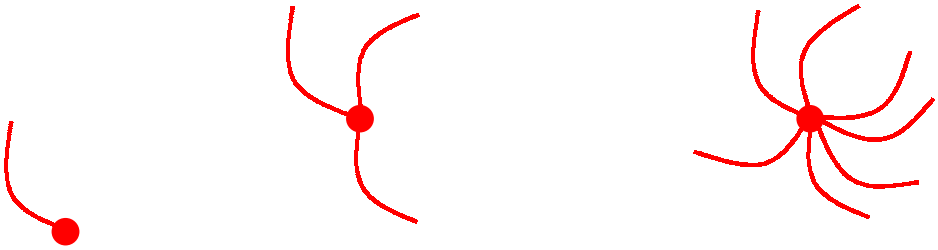
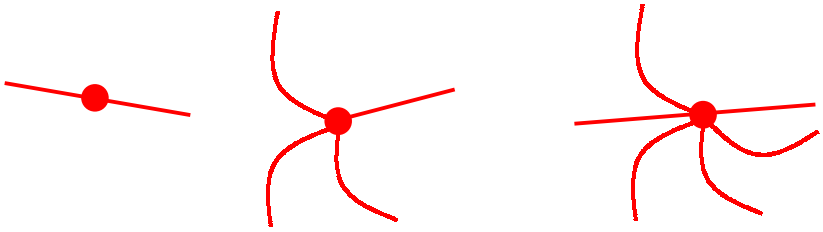
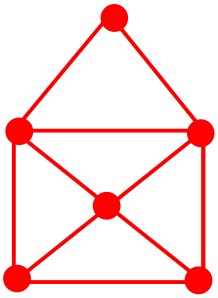
These are odd nodes. Can you see why?



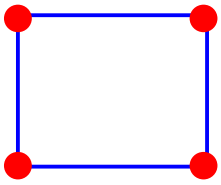
These are even nodes. Can you see why?

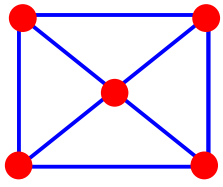


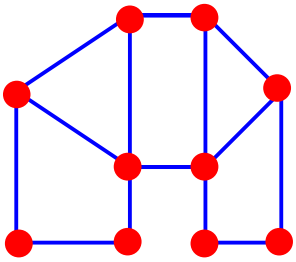
How many odd nodes does this shape have?
How many even nodes does it have?



How many odd and even nodes do these shapes have?







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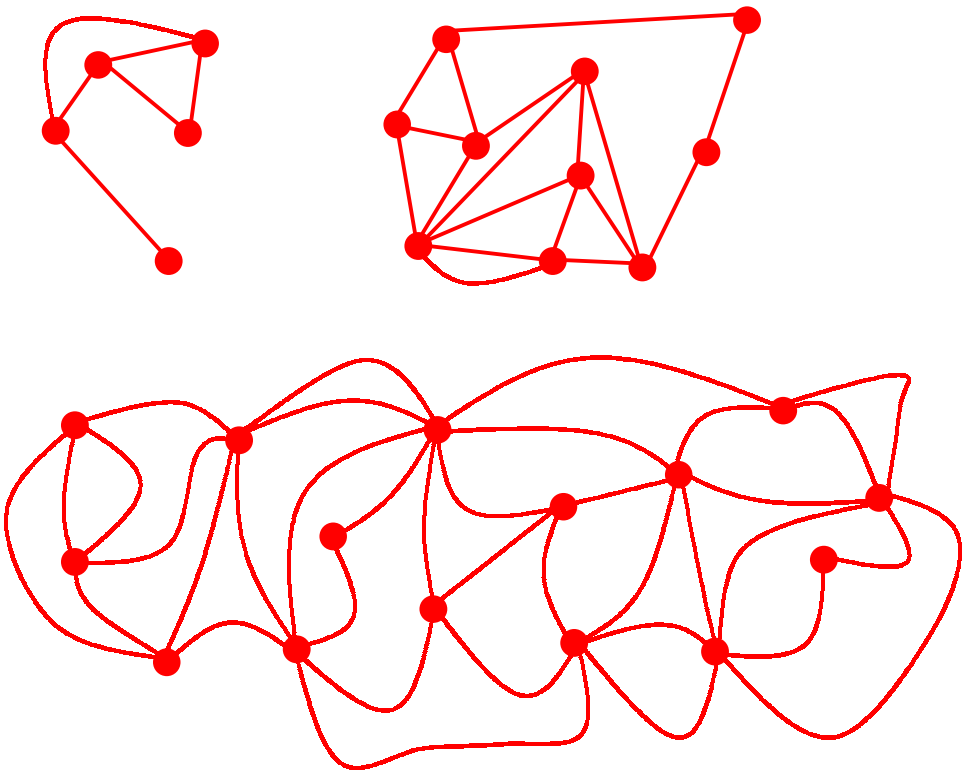
The Problem

Draw some networks of your own.
Count how many odd nodes and even nodes each network has.
Record you results in a table.

Can you find a rule that tells you if it is possible to draw a shape without taking your pencil off the paper and without going over the same line twice?

When you have done this, you should be able to say whether you can draw the following shapes without even trying them!!!! Brilliant!



Good Advice:

Work carefully. It is important that you check your work. If you make a mistake, you will not be able to discover any rules.

Answer Guide

Here are some possible answers and notes for guidance.

Odd nodes obviously have an odd number of arcs going into them and even nodes have an even number of arcs.

Surprisingly, there is a set of simple rules for determining whether or not you can draw a network without taking your pencil off the page and without going over the same line twice:

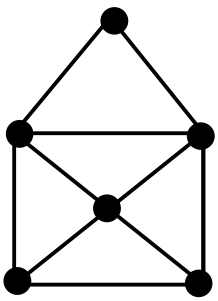
If all the nodes are even, the shape can always be drawn and you can start anywhere you like.

If exactly two of the nodes are odd and all the others are even, it is possible to draw the network, but you must start at one of the odd nodes (you will inevitably finish at the other).

If there are four or more odd nodes, the network cannot be drawn no matter where you begin.

N.B. There are always an even number of odd nodes.
This must be the case, since each arc has two ends.
Therefore the total number of ends must be even.
Since even nodes always use an even number of ends, the odd nodes must use an even number of ends too.
Since there are an odd number of ends at each odd node, the number of odd nodes must be even. Piece of cake!

Eg. Look at this network:



There are ten arcs. This means there are twenty ends of arcs (an even number, no matter how many arcs there are)
There are four even nodes using, in this case, fourteen ends of arcs. This is an even number. Subtract fourteen from twenty equals six. This must be even since we are always subtracting an even number from an even number at this point.

There are three ends of arcs at each odd node (an odd number, naturally), so the number of odd nodes is six divided by three which is two (this is always even).
Even if you move one end of an arc from one node to another, the total number of odd nodes is always even!

Answer Guide (Contd)

A shape that has no odd nodes can be drawn starting anywhere because each node has an even number of arcs coming into it and so, when you enter a node, you can always get out again. If there are odd nodes, sooner or later you will become trapped, because the last time you enter that node, you will have run out of lines to escape. Therefore you cannot draw the shape unless it has just two odd nodes - one for the beginning of the line and one for the end. Try it and see.

In doing this investigation, you need to make sure that the children draw a number of networks with no odd nodes, a number with two odd nodes and a number with more than two odd nodes.

If you wish to extend this investigation, you could ask the children to look at how many 1-nodes, 2-nodes, 3-nodes etc there are in each network and see if they can find any rules. (a 3-node is obviously a node with three arcs going to it).

We do not normally count the number of 2-nodes, because every point on a line is a 2-node, but if you just count corners as two nodes, it will not make any real difference to the investigation.