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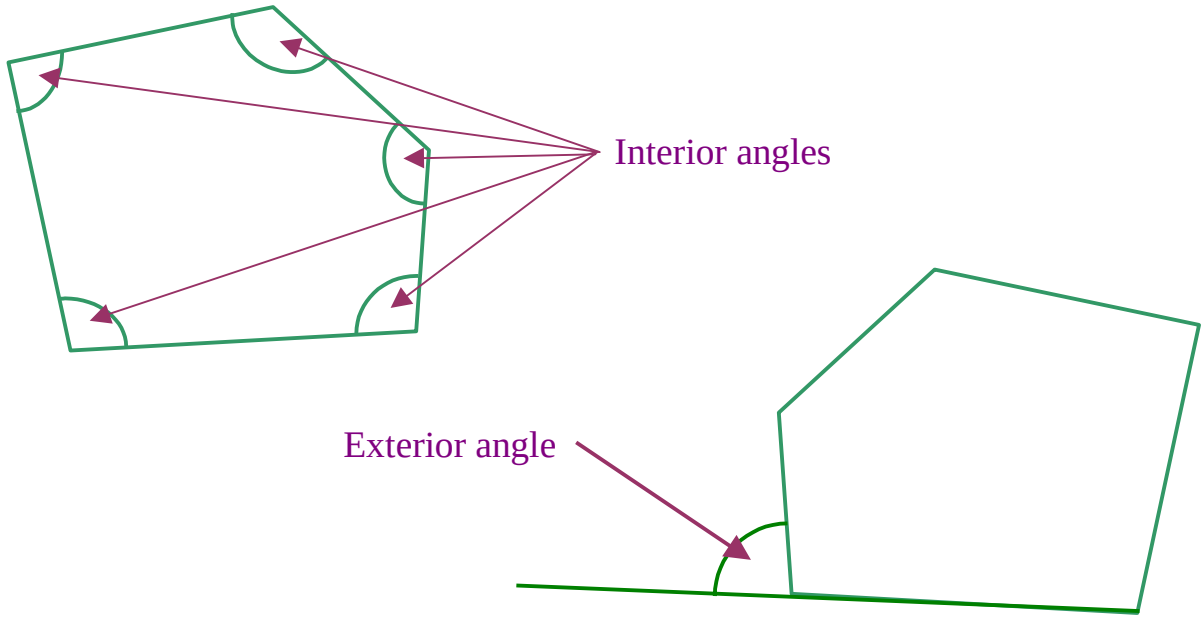
INVESTIGATION

Exterior and Interior Angles of Polygons

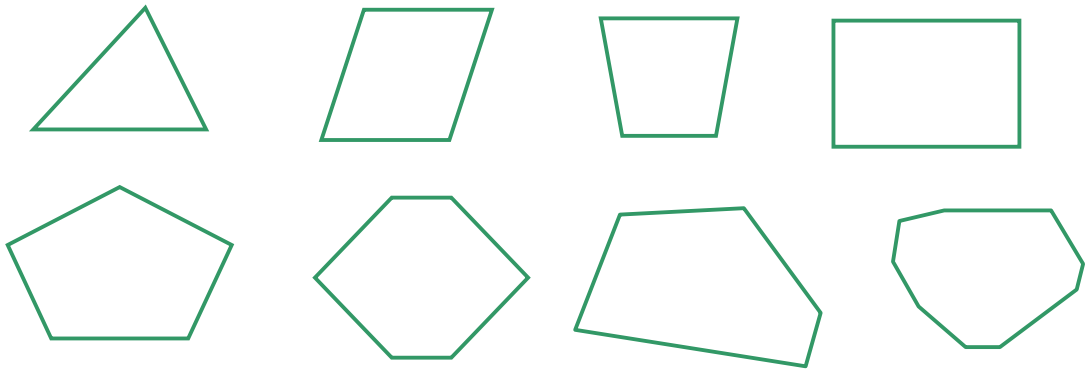


MathSphere

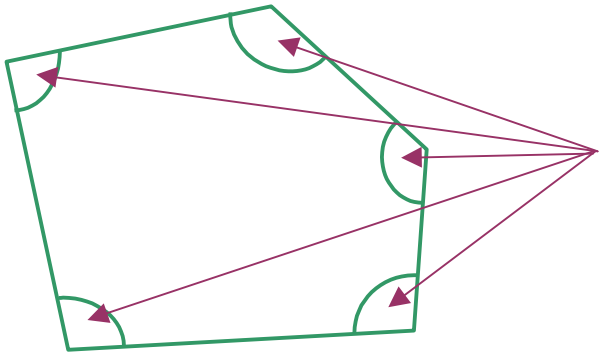
Exterior and Interior Angles of Polygons

First some definitions!!!!

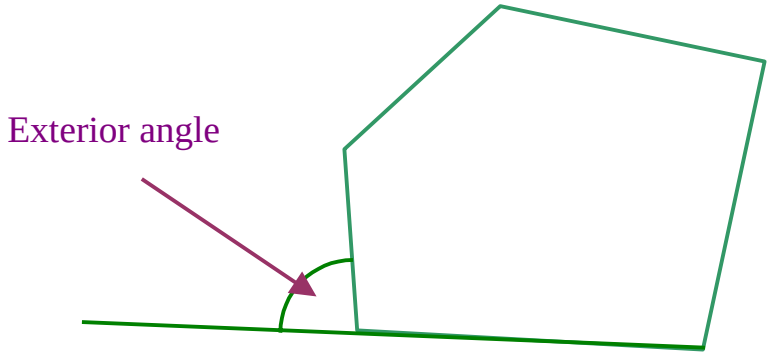
A polygon is a shape with three or more straight sides such as triangles, quadrilaterals, pentagons. These are all polygons:



An interior angle is an angle inside a polygon:



An exterior angle is the angle on the outside of a polygon between one of the sides and an adjacent side that has been extended:



The Problem

Your task is to investigate interior and exterior angles of different polygons. See what rules you can discover for different polygons and what patterns you can find.

Good Advice:

Make sure you know how to use a protractor or other angle measuring device.

Work very carefully and measure all the angles as accurately as you can.

Use a good ruler and a very sharp pencil to draw the shapes.

Work in a methodical way, recording your results carefully as you go.

Some ideas:

Begin with an easy shape, such as triangles. Measure the interior angles of several triangles and see if you can discover something about them.

Then try quadrilaterals. Then pentagons and so on.

Do not jump about between triangles, rectangles and pentagons - you will get very confused.

Then look at exterior angles of triangles, quadrilaterals etc.

Try to find as many rules and patterns as you can.

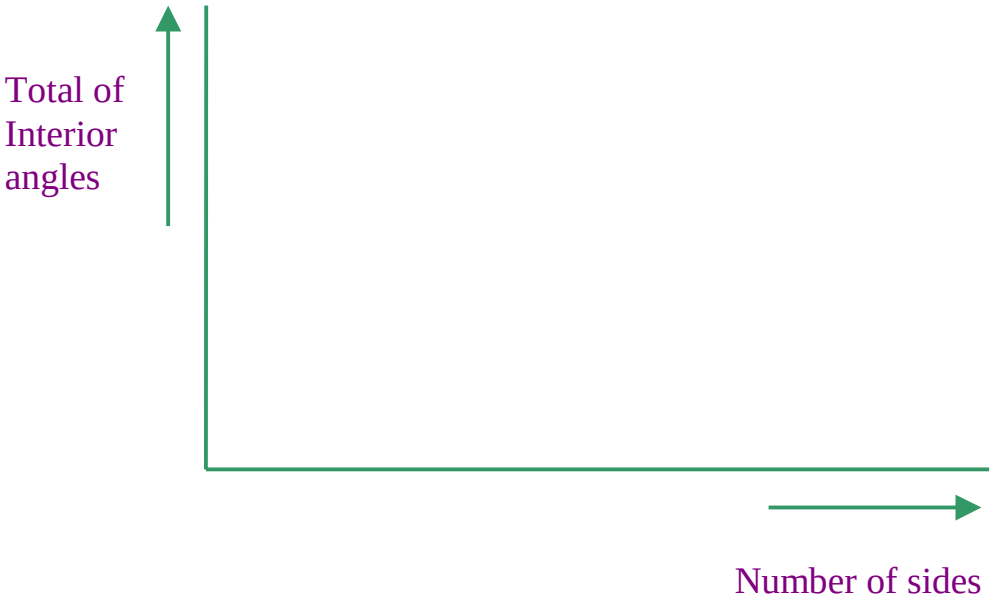
Some more ideas:

Record your results in tables. One like this may be helpful:

Shape	First angle	Second angle	Third angle	Total
Triangle A	52^0	48^0	81^0	181^o
Triangle B				
Triangle C				
Triangle D				
Triangle E				

When you know something about the interior angles of polygons, you could perhaps draw a graph of your results.

On the horizontal axis, you could put the number of sides of the polygons and on the vertical axis put the total of all the interior angles.



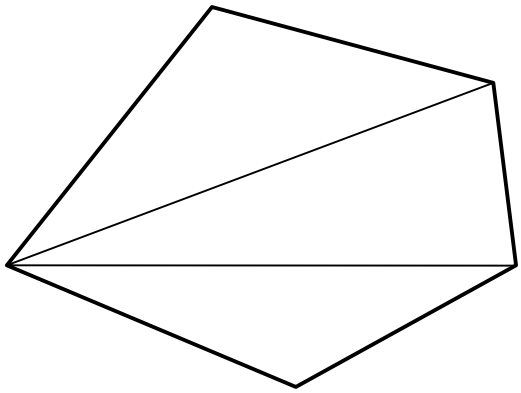
Do not be afraid to try ideas of your own - you never know what you may discover!!

Answer Guide

Here are some possible answers and notes for guidance.

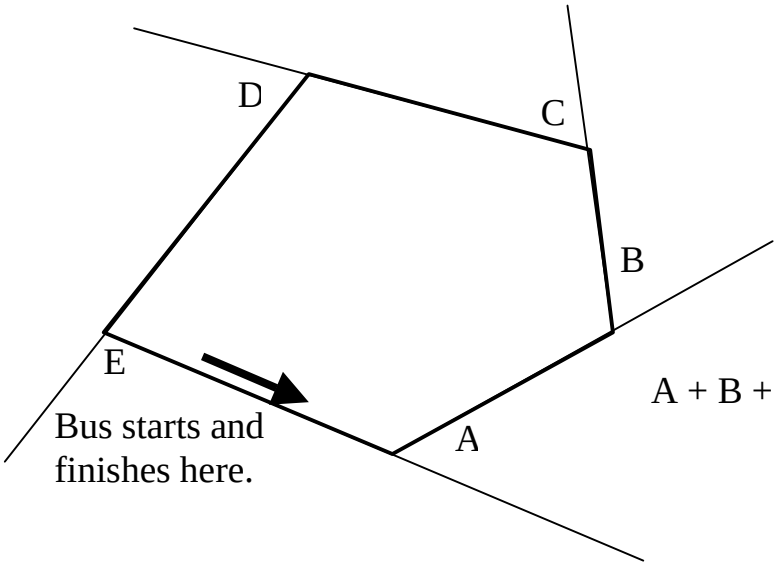
The sum of the interior angles of a triangle is 180^0, a quadrilateral 360^0, a pentagon 540^0 etc. In other words the sum of the interior angles is $(n - 2) \times 180^0$, where n is the number of sides. This is independent of whether the shape is regular or not. This makes a very nice straight line graph.

This result can be easily seen by slitting polygons into triangles by drawing diagonals from one vertex and noting that the sum of the angles in the triangles equals the sum of the angles in the polygon.
Eg.



Three triangles means
3×180^0 for a pentagon.

The sum of the exterior angles of any polygon is always 360^0, regardless of the number of sides. This can easily be seen by imagining a bus driving around the polygon and noting that it turns by the exterior angle at each vertex. The bus ends up facing the same way as when it started, therefore it must have turned through 360^0 altogether. Therefore the sum of the exterior angles is 360^0 .



$A + B + C + D + E = 360^0$

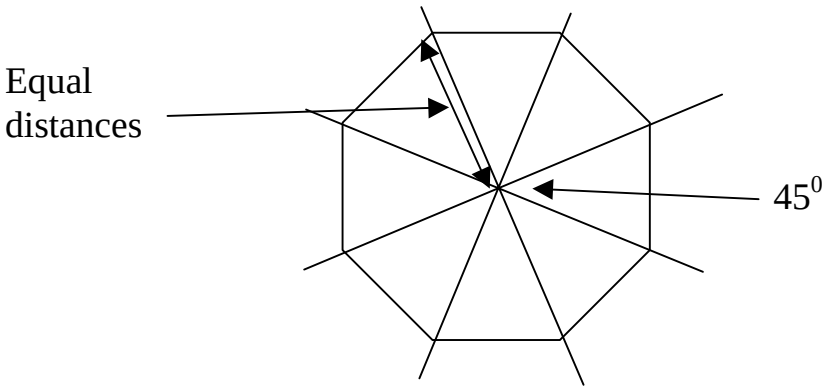
Answer Guide (Contd)

Other ideas:

Encourage (nay, insist!) children work methodically and give them some hints on how to go about this. Experience shows that many children will do, say, two different triangles and then a quadrilateral and then one or two pentagons. They will have a list of totally muddled results that mean very little. Keep them on one shape until they have discovered something worth recording and discussing. This investigation is good for encouraging methodical ways of working and recording as well as for investigating some interesting mathematics.

Make sure that they realise that there are many quadrilaterals apart from squares and rectangles. In this investigation, irregular polygons feature just as highly as regular or symmetrical ones.

Look at different ways of constructing shapes. How can one construct a regular octagon? One method often given is to start at the centre and draw eight 45^0 angles in a complete circle. Then measure the same distance along each radius and join them up.



However, when children have discovered how to find the interior angle of a regular polygon, they can pretend they are the bus driver (see above) and draw the shape using interior angles.

Encourage older/brighter children to find a method of calculating interior and exterior angles of regular polygons without drawing them. Eg. What is the interior angle of a polygon with 45 sides?

Interior angle of a polygon with n sides = $\frac{(n - 2) \times 180}{n}$

Exterior angle of a polygon with n sides = $\frac{360}{n}$