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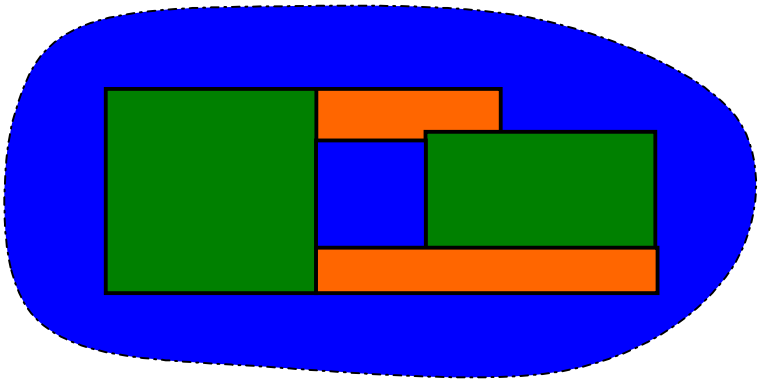
INVESTIGATION

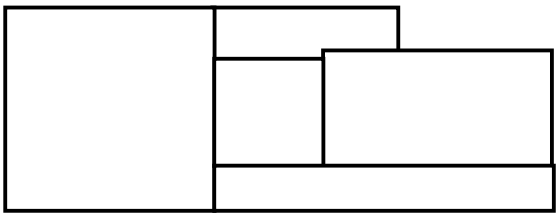
Colouring Maps

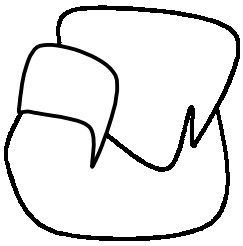


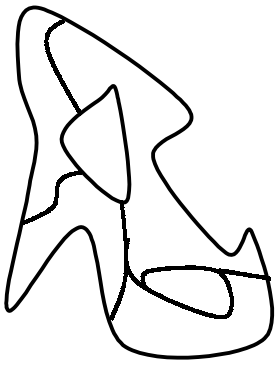
MathSphere

Colouring Maps

What is a map?
We could discuss this question for some time, but in mathematics it is easy - it is a collection of shapes that touch each other. Here are some examples of mathematical maps:





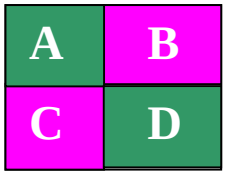


These are just like maps in an atlas. The only difference is that you won't recognise the countries!

The outside area is counted as a country in mathematics. If you have trouble with this idea think of it as the sea.

When these maps are coloured each country must have a different colour to the one next to it and the outside country (the sea must be coloured too).

If two countries meet at one point they can be the same colour.



In this map A and D can be the same colour because they only meet at one point and B and C can be the same colour for the same reason. A and B must be different colours because they touch each other along a border.

The outside must be a different colour (say, blue), because it touches all of the countries. Therefore 3 colours are needed to colour this map.

The Problem

Your task is to investigate all sorts of maps and see how many colours are needed to colour them. Don't forget the outside needs a colour too.

What is the smallest number of colours that you would need to colour any map that someone gave you.

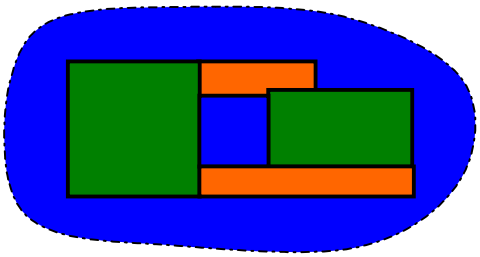
That's a very big question!

It is a big question. Let me put it another way:

Imagine you are sitting in a sealed room with a letterbox in the door. People post maps through the letterbox and you have to colour them.

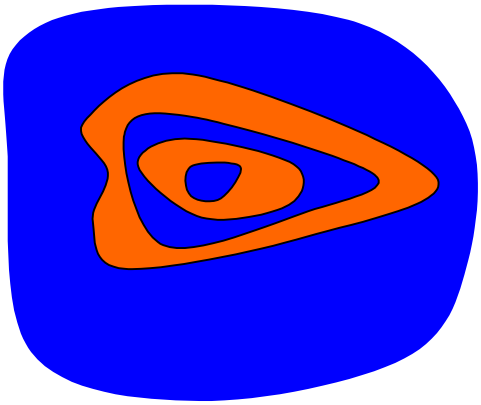
What is the smallest number of colours you would need to have to be sure you could colour any map that was posted through the door?

Here are two examples to look at:



The outside colour can be the same as the inside colour as the inside country does not touch the outside at all.

We need only three colours for this map.



In this map the outside colour can be blue, the same colour as two of the inside colours because they do not touch the outside. Therefore, only two colours are needed for this map.

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Answer Guide

This investigation relates to a very old mathematical problem in the branch of mathematics known as 'topology', ie the study of the properties of shapes in which size does not matter (unlike geometry and trigonometry).

Children can become very absorbed in this problem and some think they can solve it in a few minutes, thus bettering the efforts of some of the finest brains in the world over the last several centuries! Great fun!

The answer seems to be that four colours are needed, providing the map is drawn on a flat piece of paper. The problem is therefore normally referred to as, unsurprisingly, 'The Four Colour Problem', but this has not been mentioned earlier so as not give the children any clues. If you draw it on a torus - a ring of round cross section (like a doughnut), for example, the number needed seems to be five.

The solution to this problem has proved to be extremely difficult to elicit, but it has supposedly been solved in recent years. The problem is that the solution was produced on a computer and consists of many thousands of steps - some mathematicians think too many for any human to ever check! The plot continues.