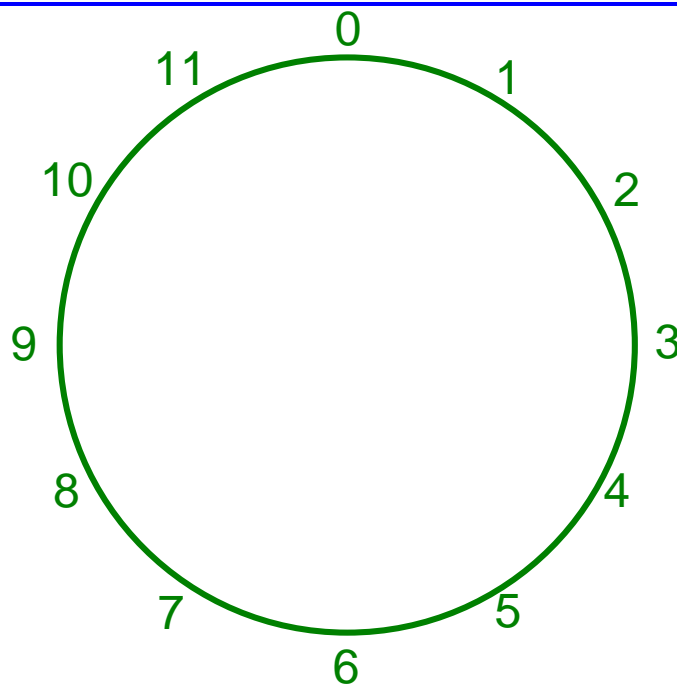




INVESTIGATION



Modulo Arithmetic



MathSphere

Modulo Arithmetic

Peter did these sums:


$$8 + 6 = 2$$

$$7 + 9 = 4$$

$$5 + 5 = 10$$

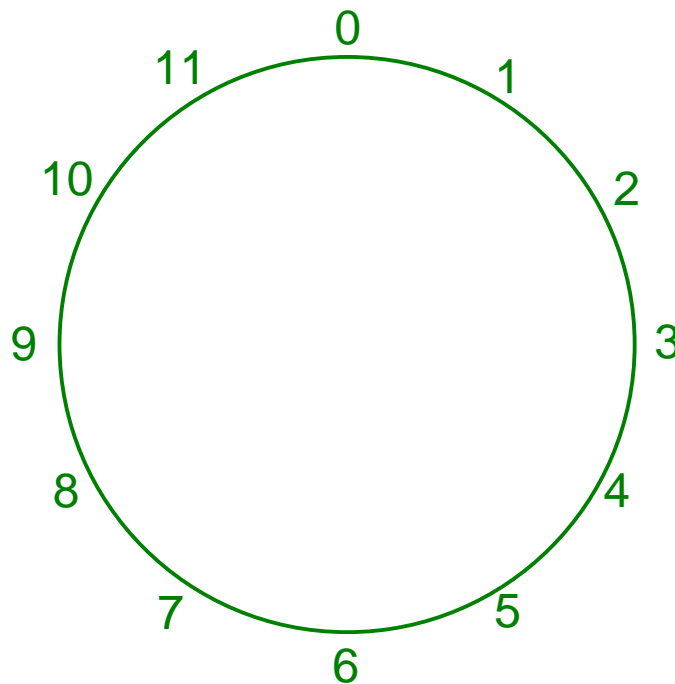
His friends thought he had gone mad, but his teacher said he had worked them out correctly.

How did Peter work out his sums?



Hint: If you have the time,
you have the answer!

In this arithmetic, we put a 0 at the top of the clock , so our clock looks like this:



Now try some sums yourself:

$7 + 6 =$

$3 + 5 =$

$8 + 11 =$

Start at 7 o'clock
and add on 6 hours.

Make up an addition table in Modulo 12. (Notice Modulo 12 uses the numbers from 0 to 11. The number 12 is not used)

[illegible]

What patterns do you notice?

Suppose you had 7 numbers on the clock (0, 1, 2, 3, 4, 5, 6).
(Notice number 7 is not used).

This would be Modulo 7 arithmetic.

Can you draw a multiplication table for Modulo 7?

What patterns do you notice?

Wouldn't it be great if you could multiply in Modulo arithmetic?

Here is an example:

Multiply 4×5 in Modulo 12.

Use the clock with twelve numbers (0 - 11)

Begin at the top (0).

Count on 4 five times.

Check you end up at 8.

This means $4 \times 5 = 8$ in Modulo 12 !

Use the same method to work out:

$$3 \times 5 =$$

$$7 \times 3 =$$

$$8 \times 4 =$$

$$6 \times 6 =$$

Wow!

Can you make up a multiplication table for Modulo 12?

Some of the answers have been worked out for you:

x	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0								
1	0	1	2	3								
2	0	2	4	6	8	10	0	2	4	6	8	10
3								9				
		etc										

What patterns do you notice?

Other Ideas to Try:

1. Try multiplying in different modulos. See what patterns you get.
2. Try making up a subtraction table. The great thing here is that you don't have any negative numbers to worry about.

Eg $5 - 11 = 6$ Try it and see!

3. What is the smallest modulo you can have?

Can you make up an addition table and a multiplication in this modulo?

Answer Guide

Normally, we would encourage all children to have a go at an investigation and they can then work at their own level. In this case, however, experience has shown that children who are not confident with their normal number work can become confused when they return to normal arithmetic operations. For this reason, we recommend that you use this investigation only with competent mathematicians. For these people it is great fun and opens up many possibilities. They generally love this type of work as they have a feeling that they are studying something not many people know about.

Things to remember:

The number of a modulo is the number of marks around the clock. These always start with 0 and the modulo number itself is not included. Eg. In modulo 8, we use the numbers 0 to 7 and 8 itself is not used. This is the only difference between a modulo clock and a real clock.

Children might like to consider what would happen if the world used modulo arithmetic all the time: I have £5 in the bank. I take out £7 and I now have £10 (Modulo 12) !!! Sounds like fun!