

## The design of the Primary National Strategy Wave 3 mathematics materials

### Guiding principles

The materials evolved as feedback was provided through pilot LEAs and in response to relevant research.

The guiding principles informing the design are:

- flexibility so that teachers can adapt them;
- sharing the purpose of each activity with the child to encourage reflection on, and ownership of, learning;
- highlighting and modelling key vocabulary throughout;
- teaching activities finishing with related activities for whole-class use, where appropriate;
- use of a variety of images and models, aiming to include some the child may not have met before;

- linking mathematics to familiar and relevant contexts;
- integrating and exemplifying mathematical problem solving;
- inclusion of games among teaching activities, possibly for sharing with parents and carers.

## Mathematical themes

The following are fundamental to the approach.

- Using and applying mathematics has been integrated. Often there are several opportunities for problem-solving within one activity, but in each, one particular opportunity has been highlighted. Aspects such as the following are incorporated:
  - encouraging children to discuss and explain in order to support development of their mathematical reasoning;
  - opportunities for children to make choices are woven into the activities, for example selecting numbers and devising calculations;
  - encouraging children's own recording to communicate mathematical thinking, focusing on efficiency;
  - opportunities for evaluating the efficiency of methods of calculation.
- Development is emphasised and key vocabulary is listed in each activity. It is important for adults to use correct mathematical language. To facilitate this, examples are given in words, for example, ' $725 \times 3$ ' is accompanied by what the adult could say to the child: 'Seven hundred and twenty-five *multiplied by* 3'.
- There is a focus on progression in counting from the earliest stages through to Year 6 to support the development of secure counting skills.
- Throughout the materials, there is emphasis on the process of estimating first, then calculating and then checking. This is denoted by the following icon:



- Decimals are addressed within meaningful contexts, for example via displays on a calculator and as a part of measure.
- Structured equipment and everyday materials are used to model mathematical concepts, supporting children's mathematical thinking and development of mental imagery. Some links to ICT resources such as the Primary National Strategy Interactive Teaching Programs (ITPs) are included.
- A wide range of resources is used in the teaching sessions. Teachers' selection of these to suit the needs of their children is an important part of adapting the materials.

## Assessment overview

The materials reflect best practice in assessment for learning as a key tool for raising achievement through:

- use of questions to elicit information about children's understanding;
- sharing the purpose of the activity with the learners;
- encouraging children's reflection on their learning and identification for themselves of possible next steps.

## Details of the materials' structure

The materials focus on a selection of the key objectives in National Numeracy Strategy *Framework for teaching mathematics*, namely, addition and subtraction, and multiplication and division objectives. Research shows that children's difficulties with calculation are highly susceptible to intervention and that individualised work with children who are falling behind in number and calculation has a significant influence on their performance.

In order **to exemplify progression in calculation, Reception, Year 2, Year 4 and Year 6 have been chosen as representative milestones**. Under each year group heading, associated knowledge and skills that contribute to understanding of the year group key objective are listed. (See first column of tracking chart.)

The whole primary age range is represented in the progression in the chart. The year group labels provide a convenient link to the National Numeracy Strategy *Framework for teaching mathematics* progression in number and calculation. As they use the chart, teachers will need to 'track back' to find the error or misconception appropriate for the child, **irrespective of the year group to which it is attributed in the progression**.

## Tracking charts

1 →

Tracking children's learning through the NNS Framework for teaching mathematics (addition and subtraction)

Year 6 key objective	Associated knowledge and skills	Errors and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Carry out column addition and subtraction of numbers involving decimals (NNS Framework for teaching mathematics, Supplement of Examples, Section 6, pages 49, 51)	Apply knowledge of the number system to enable efficient counting of a large number of objects. Add and subtract multiples of ten, a hundred and a thousand. <b>1 Y6</b>	Has inefficient counting strategies and/or insecure understanding of the number system. <b>1 Y6</b>	Imagine you have a money box containing 2p and 1p coins. What do you think would be a good way to count these quickly to find out how much money there is? What is $60 + 207 \dots 60 + 307 \dots 60 + 407$ ? What changed when you found $60 + 407$ ? What is $40 + 407 \dots 400 + 407$ ? Which answer is the larger? How is the calculation $40 + 400 + 4000$ different from the others? What is $60 - 207 \dots 600 - 2007 \dots 6000 - 20007$ ? Explain how you worked these out. What is $6000 - 2007 \dots 6000 - 207$ ?	Practical opportunities to develop efficient counting strategies for a range of objects, for example coins, cubes, counters, collectable cards, etc. Count forwards and backwards in tens, hundreds and thousands from different starting points, including starting numbers that are not multiples of ten or a hundred. Use an empty number line to support this development. Order multiples of a hundred and a thousand.	Carry out simple calculations that involve crossing the boundary from hundreds to one thousand and vice versa, supported by an empty number line and extending this to a visualised image to develop mental calculation.
	Give an estimate by rounding up, to determine whether the answer to a calculation is sensible. <b>2 Y6</b>	Rounding inaccurately, particularly when decimals are involved, and having little sense of the size of the numbers involved. <b>2 Y6</b>	Is 26 nearer to 20 or 30? Is 271 nearer 270 or 280? Is 1.6 nearer to 1 or 2? Draw a sketch to illustrate your answer and explain how you know.	Use number squares and/or number lines to consider the order and comparative value of numbers to support rounding.	Consider pairs of items from a catalogue and ask child to estimate whether a £10 (or £20, etc.) note would be enough to buy both the items?

2 →

3 →

4 →

5 →

6 →

7 →

- 1 Key objective.
- 2 This column lists associated knowledge and skills that contribute to understanding of the key objective.
- 3 Common errors and misconceptions linked to specific knowledge and skills are listed to support diagnosis of children's difficulties.
- 4 Questions in this column can be used to help the teacher decide where the child's difficulties lie.
- 5 Examples of the types of teaching activity in the A4 booklets (see below).
- 6 This column provides ideas to develop when the child has improved their understanding of the identified difficulty. The teacher can make use of these ideas to consolidate understanding and extend thinking.
- 7 Code referencing to an A4 teaching unit.

Six essential areas to support a child's learning in calculation are the basis of the Primary National Strategy *Using models and images to support mathematics teaching and learning in Years 1 to 3* (DfES 0508-2003 GCDI) and the focus on

these is reinforced in the Wave 3 mathematics pack.

These areas are:

- ordering numbers;
- counting on and back;
- partitioning and recombining;
- addition and subtraction facts within 20 (not just those that total 20);
- understanding of the four operations;
- problem-solving strategies.

## A4 booklets – teaching units

The structure of each booklet is as follows:

- focus error/misconception;
- opening teaching activity addressing error/misconception;
- a number of Spotlights (short focused teaching activities from which to select);
- final Spotlight, which includes assessment opportunities, often encompassed in a game, key vocabulary checklist, and intended learning outcomes list.

### Opening teaching activity

### Spotlight

The diagram illustrates the structure of an A4 booklet, showing how various components are labeled and linked to specific parts of the 'Opening teaching activity' and 'Spotlight 2' pages.

**Labels and their corresponding parts:**

- Error/misconception heading:** Points to the heading 'Has insecure understanding of the structure of the number system, resulting in addition and subtraction errors and difficulty with estimating' on the Opening teaching activity page.
- Problem-solving emphasis:** Points to the 'Problem-solving emphasis' section on the Spotlight 2 page.
- Suggested time:** Points to the 'Time 10-15 minutes' label on the Spotlight 2 page.
- Activity title (for Spotlights):** Points to the 'Spotlight 2' title on the Spotlight 2 page.
- Key vocabulary:** Points to the 'Key vocabulary' section on both the Opening teaching activity and Spotlight 2 pages.
- Resources:** Points to the 'Resources' section on the Opening teaching activity page.
- Teaching activity:** Points to the 'Teaching activity' section on the Opening teaching activity page.

**Opening teaching activity page details:**

- Has insecure understanding of the structure of the number system, resulting in addition and subtraction errors and difficulty with estimating**
- Opportunity for: developing mental images**
- Resources:**
  - Two sets of number cards (0-9) (Resource sheet 1)
  - 100-squares
  - Place value blocks
  - Long number line (or number line)
  - Story cubes
  - Bundles of sticks or other Base 10 equipment
  - To come for counting
- Key vocabulary:** digit, larger/smaller, estimate, guess, nearest/furthest, more than, less than, units, ones/tens/hundreds, rounding to the nearest
- Time:** 10-15 minutes
- Teaching activity:** We are going to be working with numbers today, decide which numbers are larger or smaller and we are going to count on a long number line. This work will help you with estimating and with doing more calculations.
- 1. Lay out the two sets of number cards (0-9) on the table.**
- 2. Can you make the numbers thirty-three and thirty-four?**
- Support the child to make:**
- 3. Which is the larger number, forty-three or thirty-four?**
- If the child knows which is larger, move on.**
- If the child doesn't know which is larger, you could count with the child using 100 on a 100-squares, pointing out the fifties and the forties.**
- Then change the numbers to 40 and 30.**
- Count in tens with the child on the 100-squares, establishing that if you had forty sweets you would have more than thirty sweets because forty is more than thirty.**
- Then you will need to make some more two-digit numbers and repeat the activity before you move on.**
- Note:** If the child seems to have problems crossing boundaries, see 1.12. If the child seems unsure of numbers, you might want to check that the child can count a large pile of cubes so that you can assess their counting skills (see also 1.15.1).
- Display a long number line, say up to four hundred. (You could stick masking tape on the floor or wall and write the numbers along it. You will need to use the number line throughout this set of activities.) Ask the child to position the numbers on the line with sticky notes or paper.**

**Spotlight 2 page details:**

- Spotlight 2**
- Has insecure understanding of the structure of the number system, resulting in addition and subtraction errors and difficulty with estimating**
- Opportunity for: focusing about numbers**
- Place value on the calculator**
- Resources:**
  - Calculator each
  - Large paper or other screen
- Key vocabulary:** digit, ones/tens/hundreds, order, before, after, rounding to the nearest ten/hundred, less than, more than, units, ones/tens/hundreds
- Time:** 10-15 minutes
- Teaching activity:** Today we are going to do some more work on place value and where numbers go on the number line. We will perform a few thinking about the errors that we see. So if you see a zero, tell me and I will record that for later.
- 1. Can you see any numbers with zeros on the number line?**
- Follow up:** Ask the child to give experience of reading numbers such as three hundred and six.
- Prop up a large book so that you can enter numbers secretly.**
- 2. How many times to key in three hundred and forty-two on your calculator and tell it the same on mine.**
- How many times to key in three hundred and forty-two on your calculator and tell it the same on mine?**
- 3. What do you think I did to three hundred and forty-two to get to that number?**
- If the child isn't sure you will need to do some further adding and subtracting of one-digit numbers.**
- 4. Can you make your number the same as mine in one move? (Showing an operation key, a number and the equals key)**
- 5. What did you do to make your number the same as mine?**
- Repeat until the child understands how to add and subtract single-digit numbers.**
- Record any numbers you use that have a zero in them.**
- Then subtract all the tens from the number secretly.**
- Show the child your screen.**
- 6. What subtraction will take away the four? (Signal to the child that you have a number with a zero in and record that for later use.)**

Specific icons are used to improve access to the text:

### Icons

 **Questions are incorporated for teachers to select from and add their own as appropriate.**



Whole-class follow-on activity.



Symbol reminding of the necessity to estimate, calculate, then check.



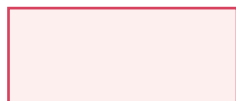
This variation of the game is harder.



This variation of the game is easier.

$12 \times 2 = 24$

Text within this symbol indicates an opportunity for recording.



Text within a shaded box indicates alternative approaches for a child who is having difficulty with the activity.



Additional game at the end of some teaching units.