# Appendix 2

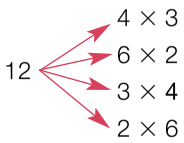
## Tracking children's learning chart – multiplication and division

The tracking charts act as an initial diagnostic tool and a background framework for the sets of teaching materials referenced to common misconceptions and errors.

Teaching unit codes are referenced against the errors and misconceptions listed in the chart, for example *1 Y6* ×/÷

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

**Year 6 key objective** Carry out long multiplication of a three-digit by a two-digit integer and short multiplication and division of whole numbers (NNS *Framework for teaching mathematics*, Supplement of Examples, Section 6, pages 67 and 69)

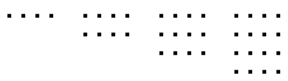
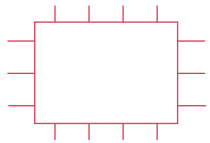
| Associated knowledge and skills | Errors and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Know by heart all multiplication facts up to 10 × 10.<br><br>Derive division facts corresponding to multiplication facts up to 10 × 10. | Refer to Year 4 chart (1 Y4 and 2 Y4) where this is described in connection with a limited range of multiplication and division facts. At Year 6 level, the number range will extend to include multiplication facts up to 10 × 10 and related division facts. | | | |
| Multiply and divide one-, two- and three-digit numbers by ten and one hundred.<br><br>Apply the associative law (but not by name) to multiply up to three-digit numbers by multiples of ten and hundred, for example:<br>147 × 20<br>= 147 × 2 × 10<br>**1 Y6** | Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example:<br>145 × 30 = 145 000<br>***1 Y6 ×/÷*** | Can you tell me a quick way of multiplying a number by one thousand?<br><br>I have thirty-seven on my calculator. What single multiplication should I key in to change it to three thousand seven hundred?<br><br>What is 3 ÷ 1? …30 ÷ 10? …300 ÷ 100? Tell me about this pattern.<br><br>If I had four thousand eight hundred on my calculator, what single division could I key in to change the display to forty-eight?<br><br>47 × 10 = 470<br>What do you think 47 × 20 equals? | Multiplication by ten, one hundred… using a calculator and recording numbers before and after multiplication in HTU columns.<br><br>Repeat for division.<br><br>Ask the child to describe patterns observed and suggest a generalisation. Ask the child to illustrate with further examples.<br><br>Demonstrate that 42 × 12 is the same as 42 × 6 × 2. What other ways can we multiply forty-two by twelve?<br><br>Repeat with other numbers. Use factor 'tree' to support this, for example:<br>  <br> | What's the best way you can think of to find:<br>71 × 20?<br>202 × 6?<br>82 × 300? |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Present the remainder in a quotient as a whole number or fraction.<br>**2 Y6** | Has difficulty, when appropriate, interpreting a remainder as a fraction, for example:<br>$16 \div 3 = 5\frac{1}{3}$<br>***2 Y6 ×/÷*** | There are twenty-six apples for four horses, so they can have six and a half each.<br>How do I know this? | What is a sensible answer for each of the following division calculations?<br>1. Divide twenty-eight eggs by six.<br>2. Divide twenty-seven bars of chocolate into four.<br>3. Divide £47 by four.<br>4. Divide sixty-two children into teams of ten. | Write a division question for your partner which you know will have a fraction as part of the answer.<br>How do you know it will have a fraction as part of the answer? |
| Dividing by numbers smaller than one, for example:<br>$12 \div \frac{1}{2} = 24$<br>$6 \div \frac{1}{3} = 18$.<br>**3 Y6** | Interprets division as sharing but not as grouping (repeated subtraction) so is unable to interpret a calculation such as $12 \div \frac{1}{2}$.<br>***3 Y6 ×/÷*** | How many half tomatoes can you get from three whole tomatoes?<br>How many quarters of pizza can you get from four pizzas?<br>Explain how to work out $5 \times \frac{1}{2}$, $5 \div \frac{1}{2}$. | Model with practical contexts and equipment, calculations such as:<br>How many halves in six? …quarters in eight? Demonstrate associated recording.<br>Use a variety of images and contexts to illustrate, for example, jumps on a number line, chocolate bars, etc. | When I divide my mystery number by half the answer is ten.<br>What's my mystery number?<br>Make up some more questions with mystery numbers. |
| Judge whether the answer to a multiplication or division calculation is reasonable.<br>**4 Y6** | Is not confident in making reasonable estimates for multiplication or division calculations.<br>***4 Y6 ×/÷*** | 600 ÷ 30 = 2<br>Can this calculation be correct?<br>How do you know?<br>Try some more, such as:<br>540 ÷ 20 = 52<br>24 × 20 = 4800<br>$15 \div \frac{1}{2} = 30$<br>$24 \div \frac{1}{4} = 22$. | Use examples of calculations to model making estimates, for example:<br>We know thirty-five divided by seven equals five. Is thirty-three divided by seven more or less than this? How can we decide?<br>Repeat using calculations such as:<br>44 ÷ 10<br>207 ÷ 20<br>Ask the child to justify the estimates they suggest. | If the question is fifty-four divided by seven, what number facts could you use to help you estimate an answer?<br>Explain how each of the facts you suggest would be useful to help you make an estimate.<br>What about the question five hundred and four divided by six?<br>Choose some more questions to use. |

# Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

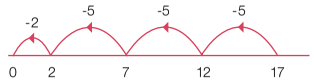
**Year 4 key objective** Use informal pencil and paper methods to support, record or explain multiplications and divisions, develop and refine written methods for TU × U, TU ÷ U, and find remainders after division (NNS *Framework for teaching mathematics*, Supplement of Examples, section 6, pages 56, 66, 68)

| **Associated knowledge and skills** | **Errors and misconceptions** | **Questions to identify errors and misconceptions** | **Teaching to address the errors and misconceptions** | **Next steps in moving towards the key objective** |
|---|---|---|---|---|
| Know multiplication facts for 2, 3, 4, 5 and 10 times tables.<br>**1 Y4** | Is not confident in recalling multiplication facts.<br>***1 Y4 ×/÷*** | What is three multiplied by four? Draw me a diagram to show what this means.<br><br>How do you know the answer to calculations such as three multiplied by zero and ten multiplied by zero?<br><br>What is four multiplied by one? How do you know? | Provide models and images to illustrate the multiplication facts, for example multiple groups of beads on a bead string, jumps along a number line.<br><br>Use relevant rows from a multiplication square as a resource on which to focus questions about the multiplication facts connected to particular numbers, illustrated with a growing array as a connected image, for example:<br><br>4 \| 8 \| 12 \| 16<br><br>  |  <br><br>Cover a criss-cross pattern with a rectangle of card or paper. Ask the child to imagine the crossovers (knots, perhaps). How many lines across? How many crossovers (knots) in each? What multiplication fact will tell you how many crossovers are under the paper?<br><br>Ask the child to make up hidden crossover patterns of their own and say the accompanying multiplication facts. |
| Know the division facts which correspond with multiplication facts listed above.<br><br>Understand the inverse relationship connecting multiplication and division.<br>**2 Y4** | Is muddled about the correspondence between multiplication and division facts, recording, for example:<br>3 × 5 = 15<br>so 5 ÷ 15 = 3<br><br>***2 Y4 ×/÷*** | Tell me the multiplication facts this four by five array shows (support this question with a diagram of the array). And the division facts?<br><br>If you know three multiplied by five equals fifteen, what number sentences using division do you also know?<br><br>Draw a picture for three multiplied by six.<br>What other multiplication and division facts can you find from the picture? | Show the child a three row by five column array, made from counters laid out. Demonstrate the multiplication fact three multiplied by five equals fifteen from the pattern:<br><br>  3 × 5 = 15<br><br>How could I explain this pattern as a division fact?<br><br>Show me from the counter pattern how you know your division fact? | Working with the child, set out thirty-five counters in a five by seven array. Ask the child to explain how the array can be seen to represent two multiplication and two division calculations.<br><br>Ask the child to list the calculations. |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Understand the effect of multiplying whole numbers less than a thousand by ten.<br>**3 Y4** | Describes the operation of multiplying by ten as 'adding a nought'.<br>***3 Y4* ×/÷** | What is the answer to forty-six multiplied by ten? …three hundred and fifty-one multiplied by ten? How do you know? | Multiply by ten using Base 10 apparatus on a ThHTU board. For example, put forty-six on the board (four tens, six ones) and label with digit cards. Multiply each piece by ten, move the pieces into the correct positions to become four hundreds, six tens, zero units and label the resulting number with digit cards. Repeat for other two-digit numbers.<br><br>Use place value (arrow) cards, multiplying the number on each place value card by ten. Replace each card by the multiplied by ten version. Bring the cards together to make the completed multiplication. | Extend this activity to work with three-digit numbers multiplied by ten, and then two-digit numbers multiplied by ten then multiplied by ten again.<br><br>Encourage children to record the calculations and see the pattern as follows:<br>46 × 10 = 460<br>460 × 10 = 4600<br>4600 × 10 = 46000 |
| Can apply the distributive law (but not by name) to multiplying, using partitioning and recombining, for example:<br>14 × 3 = (10 × 3) + (4 × 3)= 30 + 12 = 42, so<br>× \| 10 \| 4<br>3 \| 30 \| 12<br><br>Know how to record TU × U multiplication calculated by a partitioning method in a grid format.<br>**4 Y4** | Does not apply partitioning and recombining when multiplying, for example:<br>14 × 3 is calculated as (10 × 3) + 4 = 34, or<br>× \| 1 \| 4<br>3 \| 3 \| 12<br><br>14 × 3 = 312, confusing the value of two-digit numbers.<br>***4 Y4* ×/÷** | What number would you partition to work out twenty-seven multiplied by three? How would you recombine your calculations? Can you spot the mistake in this calculation:<br>15 × 7<br>= (10 × 7) + 5<br>= 75? | Roll a dice three times to generate a range of multiplication calculations, for example 3, 5, 6.<br><br>Record 36 × 5 to demonstrate and explain partitioning and recombining.<br><br>Ask child to make another different multiplication calculation to demonstrate and explain. Use place value (arrow) cards to demonstrate partitioning of two-digit numbers prior to multiplication and again to demonstrate recombining. | Extend to using partially completed calculations for child to complete, for example:<br>16 × 3 = (10 × ) + ( × 3)<br>= 30+<br>= 48<br><br>× \| 10 \|<br>3 \| \| 18<br><br>Encourage the child to make a partially completed multiplication calculation for a partner to solve. |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Recognise that the commutative law holds for multiplication but not for division.<br>**5 Y4** | Assumes that the commutative law holds for division also, for example, assuming that<br>15 ÷ 3 = 5, so 3 ÷ 15 = 5.<br>***5 Y4 ×/÷*** | How do you work out your answer to three divided by fifteen? Take some cubes, or use a diagram to explain what this calculation means | Use objects and a number line to work out calculations such as 16÷4, reading this as divide sixteen into groups of four. How many groups are there? Then compare this to 4÷16 reading this as divide four into groups of sixteen. How many groups are there? Highlight that this is not the same. | Provide a range of division calculations to be sorted into those with answers that are whole numbers and those involving fractions, for example,<br>10 ÷ 5 = 5 ÷ 10 =<br>16 ÷ 8 = 8 ÷ 16 = |
| Understand the idea of remainder, and when to round up or down after division.<br>**6a Y4**<br>**6b Y4**<br>**6c Y4** | Writes a remainder that is larger than the divisor, for example, 36 ÷ 7 = 4 remainder 8.<br>***6a Y4 ×/÷***<br>Discards the remainder; does not understand its significance.<br>***6b Y4 ×/÷***<br>Does not recognise when a remainder is significant in the decision about whether to round up or down.<br>***6c Y4 ×/÷*** | With 29p to spend, you want to buy as many sweets at 3p each as you can afford? How many sweets can you buy? Show your working out on a number line.<br>Which of these calculations is correct?<br>36 ÷ 7 = 4 remainder 8<br>36 ÷ 7 = 3 remainder 15<br>36 ÷ 7 = 5 remainder 1<br>36 ÷ 7 = 5.<br>Which do you think is the best one? Why?<br>There are thirty-two children and each tent takes three children. What is the least number of tents they will need? | Use a number line to highlight the idea of a remainder. Demonstrate repeated subtraction by 'stepping back', for example, 17 ÷ 5 can be represented by:<br>  <br><br>Provide opportunities to consider how results of calculations may be rounded up or down in order to make sense of the context. For example:<br>• How many tables are needed to seat a class of thirty-two children if five children can sit at a table? This would involve rounding up to seat all the children.<br>• How many £5 books can be bought with £32? This would involve rounding down to ensure enough money to pay for the purchase. | Extend to the context of money, weight and length to demonstrate how the remainder can be subdivided into smaller units. For example, £60 shared equally between eight. The remainder of £4 can be divided further to give an answer of £7.50 each. |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Know how to record division as repeated subtraction, with appropriate use of chunking.<br>**7 Y4** | Continues to subtract twos when calculating twenty divided by two without using knowledge that two multiplied by five equals ten.<br>***7 Y4* ×/÷** | Ask the child to look back at a division calculation they have just completed. Can you think of a quicker way to find out an answer for this question? | Use a bead string to illustrate:<br>• How thirty divided by two can be calculated as thirty with ten subtracted three times. Every ten is five twos. Altogether fifteen twos have been subtracted so thirty divided by two equals fifteen.<br>• How to work out seventy-six divided by five by starting from seventy-six beads, remove ten lots of five, that is fifty, and then five lots of five, that is twenty-five. Altogether how many groups of five have been slid along the bead string?<br>76<br>-50 $10 \times 5$<br>26<br>-25 $5 \times 5$<br>1<br>so 76 ÷ 5 = 15 remainder 1 | Ask the child to work out fifty-three divided by four using knowledge of the four times table, and to record this as repeated subtraction using not more than three steps. |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

**Year 2 key objective** Understand the operation of multiplication as repeated addition or as describing an array, and begin to understand division as grouping or sharing; know by heart facts for the 2 and 10 multiplication tables; and know and use halving as the inverse of doubling. (NNS *Framework for teaching mathematics*, Supplement of Examples, section 5, pages 47, 49, 53, 57)

| **Associated knowledge and skills** | **Errors and misconceptions** | **Questions to identify errors and misconceptions** | **Teaching to address the errors and misconceptions** | **Next steps in moving towards the key objective** |
|---|---|---|---|---|
| Carry out repeated addition, recognise the relationship between multiplication and repeated addition, and use the associated vocabulary of multiplication.<br>**1 Y2** | Still counts in ones to find how many there are in a collection of equal groups; does not understand vocabulary, for example, 'groups of', 'multiplied by'.<br>***1 Y2* ×/÷** | Show an array of six rows of two.<br><br>How many sets of two can you see? How many are there altogether? Repeat with six rows of ten. | Reveal an array of twos, one row at a time. Encourage the child to imagine the next row and to say how many they will see altogether when that row is revealed. Repeat with pairs of counters set out along a number line. Encourage the child to step over the pairs with their finger as they count up in twos. How many twos? What calculation have we just done? | Encourage counting aloud in twos and tens using images and resources such as number lines, counters and arrays.<br><br>Make the connection between the multiplication facts written as a table:<br>$2 \times 1 = 2$<br>$2 \times 2 = 4$, etc. |
| Multiply two or ten by a single-digit number, by counting up in twos and tens from zero.<br>**2 Y2** | Does not link counting up in equal steps to the operation of multiplication; does not use the vocabulary associated with multiplication.<br>***2 Y2* ×/÷** | How can you quickly work out two multiplied by seven?<br><br>What steps could you count up in to help you?<br><br>How many steps do we need? | Using seven paper strips each showing two dots, make an array seven rows by two columns. With the child, arrange the strips into a 'number line' of seven pairs of dots. Count up in twos to fourteen. Count the steps and record $2 \times 7 = 14$. Rearrange the strips back into the seven by two array and emphasise the link to two multiplied by seven equals fourteen. Add and remove strips and repeat. | Display the number statement<br>$10 \times 6 = ?$<br><br>Ask the child to use paper strips with ten dots on each, to form an array that represents ten multiplied by six.<br><br>How many dots are there?<br><br>What if you were to arrange the strips in a number line? How many dots would there be? How do you know?<br><br>Ask the child to choose a different array. |

# Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Interpret an array as a repeated addition and as a multiplication, and recognise how the array can be described as, for example, 3 + 3 + 3 + 3, 3 × 4, or as 4 + 4 + 4, 4 × 3.<br>**3 Y2** | Does not focus on 'rows of' or 'columns of', but only sees an array as a collection of ones.<br>***3 Y2* ×/÷** | Display six rows of two. How many rows are there? How many dots are there in each? How can you describe the array?<br><br>Turn the array through 90°. How has the array changed? Can you describe the array to me now?<br>What's the same and what's different? | Display the first row of a four-by-five array. Here are five dots. There are four rows, which are exactly the same. Make the array with these counters. Tell me about your array. What numbers could you add together to find the total? What numbers could you multiply? | Reveal the first row and first column of a four-by-five array of dots.<br>  <br>Describe this array to me. Write a number sentence to match the array.<br><br>Make up a number sentence that matches another array and draw the array. |
| Recognise that doubling and multiplying by two are the same, and use known multiplication facts and partitioning to double numbers to fifteen.<br>**4a Y2**<br>**4b Y2** | Has difficulty relating multiplying by two to known facts about doubles; records double four as 4 + 4.<br>***4a Y2* ×/÷**<br><br>Does not use partitioning to find double twelve or double thirty-five.<br>***4b Y2* ×/÷** | What is six multiplied by two? How did you work it out? What is double six? What did you notice?<br><br>What is the answer to double ten? …double four?<br>How can we use these answers to find double fourteen?<br><br>How can we work out double thirty five? | Use a bead string to make a set of five beads. Ask what double this set of beads would be. Make another set of five beads. Use the vocabulary double five beads, five beads twice, five beads multiplied by two. Agree that there are ten beads in each case and record: Double 5 = 10 and 5 × 2 = 10.<br><br>Use the beads to double ten.<br><br>Make a set of fifteen beads. Separate into ten and five and double the ten and five. Combine to make thirty. Record:<br>Double 15 = Double 10 + Double 5<br>= 20 + 10<br>= 30.<br><br>Repeat for other two-digit numbers. | Illustrate the following with Base apparatus and ask the child to continue these patterns, using partitioning as appropriate.<br><br>2 + 2 = 4<br>3 + 3 = 6<br>4 + 4 = 8, etc.<br><br>10 + 10 = 20<br>11 + 11 = 22<br>12 + 12 = 24<br>13 + 13 = 26, etc.<br><br>20 + 20 = 40<br>25 + 25 = 50<br>30 + 30 = 60<br>35 + 35 = 70, etc.<br><br>Use appropriate vocabulary as you record these patterns as multiplications, for example:<br>3 × 2 = 6<br>13 × 2 = 26<br>25 × 2 = 50 |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Recognises that when finding the double of a number, half the answer is the original number. Uses the inverse of double to find halves of small even numbers to ten, using known facts.<br>**5 Y2** | Does not use knowledge of doubles to find half of a number; for example, continues to find half by sharing, using a 'one for you' approach and cannot apply knowledge of doubles.<br>***5 Y2 ×/÷*** | What do you think half of ten is? How do you know?<br><br>Can you think of a number that's easy to halve? Why do you think it's easy to halve your number? | Tell the child you have three counters in your left hand and three in your right hand. How many counters have I altogether? Give the child eight counters. You have double the counters I have in my hand, how many have I got?<br>Show four numbers. What is double four? What is half eight? etc. | I think of a number and double it and my answer is eight. What number was I thinking of? |
| Share a given number of objects out equally, recognise the relationship between sharing equally and division and use the vocabulary of division to describe the process, for example 'divide by', 'share equally'.<br>**6 Y2** | Is not systematic when sharing into equal groups, using a 'one for you' approach; does not use the language of division to describe the process.<br>***6 Y2 ×/÷*** | Here are twelve counters, share them out equally into these three boxes. How many counters are there in each box? Here are six coins, can you divide them between three purses? How many will be in each purse?<br><br>Altogether we had eighteen counters and there are six in each box, can you describe what we have done? | Share a set of picture cards equally between a number of players. Describe the process using the correct language of division, for example 'twenty cards divided between four players gives each player five cards'. | At the camp there are six tents. The children wonder how many will be sleeping in each tent. They know that there are eighteen children in the group, and there has to be the same number in each tent. How many sleep in each tent? |
| Begin to understand division as repeated subtraction or grouping.<br>**7 Y2** | Does not understand that 'sets of' or 'groups of' need to be subtracted to solve the problem.<br>***7 Y2 ×/÷*** | There are fourteen children and the children are asked to work in pairs (twos). How many pairs are there? | Show the child twelve cubes. Ask them to take two cubes and make a tower. Repeat to make another tower and again to make a third tower. Model the language at each stage, for example 'four cubes makes two towers'. Then ask the child how many towers of two they can make using twelve cubes, showing the array of two rows by six columns so that the they can relate the problem to a visual image. | Invite the child to use twelve cubes to make towers of three and ask them to explain how many towers can be made. Repeat with the towers of four and six.<br><br>Ask the child to chose an even number of cubes and predict how many towers of two they would be able to make. Get them to check their answer against their prediction and reflect on what they notice. |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

**Reception key objective** Use developing mathematical ideas and methods to solve practical problems (NNS *Framework for teaching mathematics*,Supplement of Examples, section 4, page 20)

| **Associated knowledge and skills** | **Errors and misconceptions** | **Questions to identify errors and misconceptions** | **Teaching to address the errors and misconceptions** | **Next steps in moving towards the key objective** |
|---|---|---|---|---|
| Count in twos and talk about how many pairs.<br>**1 YR** | Confuses numbers when counting in twos; has difficulty understanding a pair consists of two objects.<br>***1 YR*** ***×/÷*** | Can you match this sock/glove to make a pair?<br>Can you find some more pairs? How many pairs have you made? | Find matching pairs from a collection of pictures or objects, introducing the vocabulary within the context of the activity and encouraging the child to use the correct vocabulary. | Play snap/pelmanism and count in twos to see how many pairs of cards you have made.<br><br>How many pairs of eyes on these three faces? If you draw four faces how many pairs of eyes would you need? How many eyes? What if you draw…?<br><br>Extend to non-matching pairs, for example pairs of children. |
| Say how many altogether in a double; recognise that finding a double means forming a pair or adding a number to itself.<br>**2 YR** | Has difficulty with identifying doubles and adding a small number to itself, for example 2 + 2, to make twice as many.<br>***2 YR*** ***×/÷*** | Within role play, engage with child and ask questions, for example:<br>If there are two wheels on this scooter, how many wheels on two scooters?<br>How many gloves make a pair? | Show one animal/object and add a second one (double it). Stress vocabulary and encourage child to explain what you are doing. | Extend to a range of doubles contexts, for example:<br><br>Put four cows in the field, now put double that number in the next field. How many altogether? What is double four? What if you put five cows in the field this time and double that number in the next field…? |

## Tracking children's learning through the *NNS Framework for teaching mathematics* (multiplication and division)

| Associated knowledge skills | Errors and and misconceptions | Questions to identify errors and misconceptions | Teaching to address the errors and misconceptions | Next steps in moving towards the key objective |
|---|---|---|---|---|
| Solve simple problems (where there are no remainders) that involve counting the numbers of objects by organising them into groups of equal size, including more than two groups; talk about the number of groups and objects, and record their answers in their own way.<br>**3 YR** | Makes unequal groups and is unable to compare the groups.<br>***3 YR* ×/÷** | Can you put the same number of spots on each wing of the ladybird?<br>Can you share out these biscuits fairly so there are the same number of biscuits on each plate? | Practical activities set in context; for example planting bulbs, giving out stickers, sharing biscuits.<br><br>Relate to stories such as *The Doorbell Rang*, by Pat Hutchins (Mulberry Books, 1986, ISBN 0 6880 9234 9) | Extend to considering groups and objects. How many plates for three dogs? How many dog biscuits do we need so that they can have three each? Can we put some marks/numbers on each plate to help us remember? What if we had twelve biscuits – how many for each dog? |
| Share objects out fairly into two or more groups, talk about whether there will be any left over.<br>**4 YR** | When sharing, can sometimes make equal groups, but has no strategies to deal with any left over.<br>***4 YR* ×/÷** | Can you share these sweets equally between you and a friend? Will there be any left over?<br><br>Can you share these marker pens between the group? Will there be any left over? What will you do with any left over ones? | Practical sharing activities related to different contexts; for example sharing classroom resources, making groups for PE. | Extend by linking to making simple predictions. Can you share these seven cubes equally into two groups? Why not? Can you choose a number of cubes that will share equally with none left over? |
| Count in tens forwards and backwards from a tens number (i.e. multiple of ten), and identify the tens number before and after a given tens number.<br>**5 YR** | Has difficulty with counting reliably in tens from a multiple of ten.<br>***5 YR* ×/÷** | Can you say all the numbers marked on this number line? For example, ten, twenty…<br>Can you put the cards in the correct order so we can count in tens to one hundred? | Provide visual resources such as a 100-square, a tens number track, large numeral cards from 10 to 100 to hold up to demonstrate counting forwards/backwards in tens from different starting numbers. Provide clues to identify particular numbers, for example:<br>Can you stand on the tens number that comes just after 60? | Extend to mental imagery using a 100-square, for example 'Look carefully at 10. Close your eyes (or look at the ceiling). Can you still see the10? Now look at the 10 and the 20. Close your eyes (or look at the ceiling). Can you see the 10 and 20? What number comes next? etc. |
| Solve problems that involve finding halves.<br>**6 YR** | When halving, makes two unequal groups or splits a single object unequally.<br>***6 YR* ×/÷** | How many sheep in the field? Can you put half of them in the pen? How many are in the pen? How many are in the field? | Practical activities with different numbers and types of objects to experience halving. Include activities involving measures; for example capacity, mass, money. | Extend to activities that involve the child in finding contexts for halving. |