Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34 or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: developing mental images

Resources

- Squared paper
- Scissors, ruler
- Cubes (some in 'ten trains')
- Place value (arrow) cards
- Glue stick

Key vocabulary

multiplied by
altogether
array
row
column
equal groups of

Teaching activity

Time 15–20 minutes

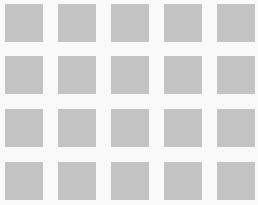
Note: It will be helpful right through this teaching sequence to have a note of the kinds of errors which the child has been making in class when doing grid multiplications. This information will be needed in Spotlights 4 and 5.)

Explain to the child that today's activity will help them to get better at multiplying numbers using a method which they have used before when they made arrays or grids, but this time using larger numbers.

'We are going to use lots of multiplying facts, including using multiplying by ten to make the multiplying easier.'

? **Can you make an array using anything on the table to show four multiplied by five?**

If the child finds this difficult, help them to make an array with cubes (ideally without joining the cubes together – just put them in rows in ones).



? **Can you describe your array to me?** *(Clarify that it is four multiplied by five, so that is five equal groups of four or, if you look at it the other way around, it is four equal groups of five.)*

? **How many cubes are there altogether? How can you be sure you are right?**

If the child starts to count in ones, stop them and ask them if they can find a quicker way, for example, counting in fours: four, eight, twelve, sixteen, twenty.

'There are twenty cubes altogether.'

? **Can you draw this array of cubes on squared paper?**
How many rows are there? *(Four)*
How many columns? *(Five)*

Record the four by five array on squared paper with a total of twenty squares and keep the cube and paper array for later.

	5
4	20

? **Do you remember doing this when we worked on finding the area of shapes?** *(Ideally remind the child about any specific activities they did.)*

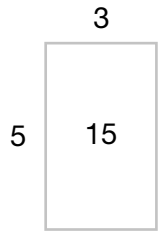
If the child seems unclear about finding the area of a rectangle, use squared paper to draw some rectangles, for example two by two and three by four. Talk the child through finding the area of these rectangles. Ask the child to write the area in the middle of each rectangle in square centimetres.

Emphasise that to find the area we must count every square.

Show how to do this for a 3 × 4 rectangle. Show how this is 3 + 3 + 3 + 3.

'Make a rectangle on squared paper four centimetres by five centimetres. We worked out that to find the area of a rectangle like this, we multiplied the length of one side by the length of the other side (4 × 5) because this is 4 + 4 + 4 + 4 + 4.'

Build up a range of rectangles on centimetre-squared paper, writing the number of rows and columns, and putting the total number of squares in the middle of the rectangle, for example five multiplied by three, six multiplied by three, as well as the four by five rectangle above. These will be used later so you might want to stick them onto plain paper.



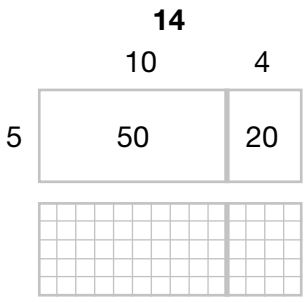
'Five multiplied by three'

It is crucial that children are familiar with writing the number of rows and columns outside the array, and the total number of squares in the middle of the rectangle. If the child seems unsure about this, do some more examples using accessible numbers, such as 2 × 4 and 2 × 6.

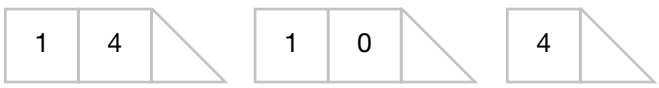
Using one of the examples that the child might manage to work with, make the number of columns ten more, for example 5 × 4 becomes 5 × 14. Ask the child to make this array with cubes, not joined together, adding ten more columns to the 5 × 4 array (adding halved 'ten trains' is quickest), while you draw it on squared paper. You might want to put the ten rods to the left of the 5 × 4.

Using a ruler, slightly separate the four original columns from the ten columns to show how the array of cubes can be split into 5 × 10 and 5 × 4 and record that on the paper array with a vertical line.

'Here is 5 × 10 and here is 5 × 4.'



Using place value cards, demonstrate how fourteen can be partitioned (or split) into ten and four.



Count the number of cubes, counting the columns, establishing that the child already knows 5 × 4 = 20.

'Look carefully and tell me the number of rows and columns in the larger part of the array.'

Check that the child understands that this array is 5 × 10, and that it is fifty cubes/squares.

? **Where shall we record the fifty?**

? **Now what can we do to find out how many cubes there are altogether in our array?**

Help the child to recombine the tens and then the ones with place value cards.

> If the child cannot see that the fifty and twenty now need to be added, show the fifty plus twenty cubes laid out in the array and count them in fives with the child.

Repeat this with another of the arrays. For example, make the 5 × 3 array into 5 × 13, again putting the new ten rods to the left of the original cubes.

Keep at least one of the larger arrays showing the partitioning into ten and a bit.

	13	
	10	3
5	50	15

? **Do you think we would get the same answer if we split the array in a different place?**

? **Can you sketch the array and we will partition it in a different way?**

? **Why do you think we split the larger arrays into ten and a bit?**

Make sure that the child sees that the thirteen could be partitioned into nine and another four, or eight and five, but by splitting it at ten and a bit we make the multiplying easier, because multiplying by ten is easier than multiplying by eight or nine.

? **What have you learned today? What would you like more practice with?**

? **Why do you think this way of multiplying works?**

Spotlight 1

Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34, or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: recognising relationships

Split them up

Time 15–20 minutes

Resources

- Squared paper
- Ruler
- Cubes
- At least two sets of place value (arrow) cards
- 100-square
- *Place value chart* (Resource sheet 39)

Key vocabulary

partition
split
recombine
add together
tens
ones
estimate

(*Note:* Keep a record of errors that the child makes during the Spotlights or classwork, so that you can reflect on them.)

Teaching activity

'Today we are going to use the place value cards to partition numbers before we multiply them. This will help you when you want to multiply larger numbers.'

Put at least two 10 and two 20 cards on the table with some 1s.

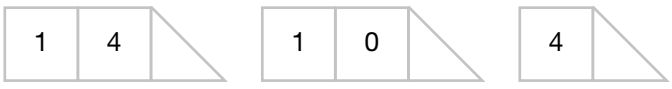
? **Can you use the place value cards to make some numbers larger than ten, but less than thirty?**

Help the child to make numbers such as fourteen and twenty-three.

? **Show me how you can partition those numbers into tens and ones.**

If the child finds this difficult, use a place value chart to show how numbers are made up of tens and ones. Find the numbers on a 100-square as well. For example, demonstrate how 24 is made up of 20 and 4.

For the rest of the lesson, you could keep just to teen numbers if you think this will help. Help the child to record some of the partitioning.



Partitions (or splits) into ten and four

? **Now can you recombine them again to make fourteen?**

When the child seems confident with the place value cards, choose one of the numbers, for example fourteen.

? **If you wanted to work out fourteen multiplied by three, which number would you partition to make the calculation easier? How can we partition fourteen?**

Sketch a grid on paper, explaining that it doesn't need to be drawn to scale, it just needs to be big enough to write the numbers clearly. Encourage the child to talk through with you how fourteen can be partitioned into ten and four, then each part can be multiplied by three.

Put appropriate place value cards on the grid.

×	10	4
3	3 0	1 2

? **Can you show me what we can do next to recombine the numbers?**

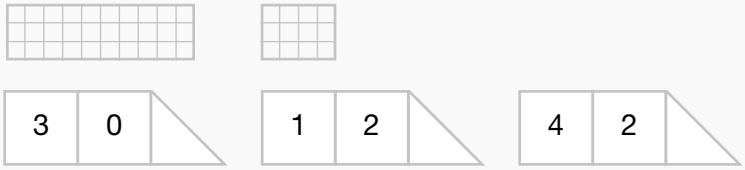
If the child is finding this hard, relate the grid and the place value cards to cubes laid out on the table.

'Three multiplied by ten is thirty and three multiplied by four is twelve.'

? **About how big is your answer going to be? About forty? Or closer to a hundred?**

? **Which numbers do we have to add together to find out how many cubes there are altogether?** *(The thirty and the twelve to make forty-two.)*

If the child is unsure about which numbers have to be added together, go back to the grid and the place value cards and relate these to the two groups of cubes, thirty cubes and twelve cubes.



? **Can you show me how you would recombine the place value cards to find the total number of cubes?**

? **How did making an estimate of the answer help you?**

If the child is unsure how to check, show how making an estimate of the calculation first can help them to see if their answer is roughly right. We can also check by doing a calculation a different way.

If you have time, repeat with other numbers, for example 13 × 5.

Encourage the child to:

- estimate the answer first;
- sketch the grid – not to scale;
- check their answer. *(Is it close to the estimate and can they check it by doing it another way?)*

? **When we are multiplying a number with two digits, what is it that we can do to make the multiplying easier?** *(Try to get at the idea of partitioning the larger number into tens and ones, and also that making an estimate first can help us to check our actual answer.)*

? **What do you think would be the most helpful way to do the partitioning for multiplying twenty-seven by three?** *(For example, would it be easier to split twenty-seven into twenty and seven or nineteen and eight? Why?)*

? **Why is it helpful to partition numbers when we multiply?**

? **On my multiplication grid, one of the totals I've put on the grid after multiplying is twenty-four. What could the numbers be that I am partitioning to multiply?**

Spotlight 2

Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34, or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: developing mental images

Number line multiplying

Time 10–20 minutes

Resources

- Squared paper
- Cubes
- Place value (arrow) cards
- *Place value chart* (Resource sheet 39)

Key vocabulary

partition, split, recombine, add together, tens, ones, multiplied by, altogether, array, row, column, equal groups of, estimate

Teaching activity

'We are going to do some more multiplying with two-digit numbers and today we are going to use a number line to help us.'

? **Can you make a two-digit number with the place value cards?**

? **Can you partition your number into tens and ones?**

If the child seems to be unsure, you can help them to partition a few more examples, but then move on.

? **Can you make the number that has one ten and three ones? What is it?**

? **Can you write the calculation thirteen multiplied by three?** *(13 × 3)*

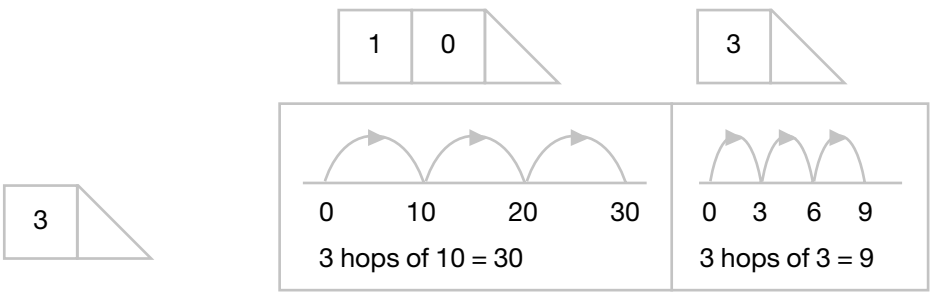
? **Make an estimate. Will the answer be about thirty or about seventy?**

? **Which number would you choose to partition to make the multiplying easier?**

Help the child to partition thirteen into ten and three.

? **Can you give me a rough estimate of what the answer is going to be? Somewhere around thirty or forty? Somewhere around a hundred? Which would be closer?**

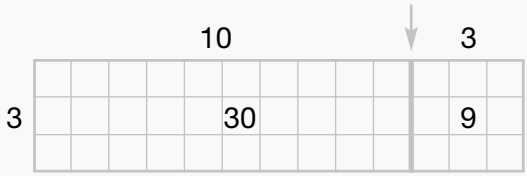
Draw a large grid, explaining that you are making it large enough to draw number lines in it. You can ask the child which numbers you need to write to the side and top of the grid, or help them to place their place value cards appropriately.



You or the child can place the number line hops.

? **Which place value cards do we need now to show the answers to the multiplication calculations we have done?** *(30 and 9)*

If the child still needs more support, you could ask them to make the array of 3 × 13 with cubes and show how that is partitioned.



? **Can you show me how you would recombine these place value cards?**

If the child cannot make the answer thirty-nine, you will need to do more examples of partitioning and recombining.

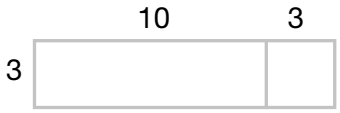
? **So what is the answer to thirteen multiplied by three?**

? **Was that quite near your estimate?**

? **Did it help you to do this calculation on a number line?**

? **If I draw a much smaller grid, can you multiply with the same numbers, 13 × 3, without a number line?**

? **How will you partition the thirteen? Can you draw in the line between the ten and the three?**



Talk the child through the positioning of the numbers.

Talk about the sizes of the two rectangles, establishing that they are 3 × 10 and 3 × 3.

? **Does it matter that our rectangles aren't exactly measured, they are just a sketch?**

Help the child to go through the grid multiplication to gain confidence in using a grid and positioning numbers appropriately.

? **Was our actual answer near our estimated answer?**

? **What helped you to learn something new today?**

Spotlight 3

Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34, or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: communicating with words, images and symbols

Three times larger

Time 10–15 minutes

Resources

- Squared paper
- Ruler
- Base 10 equipment, or bundles of straws, bead strings
- Place value (arrow) cards

Key vocabulary

partition
split
recombine
add together
tens
ones
multiplied by
altogether
array
three times larger
estimate

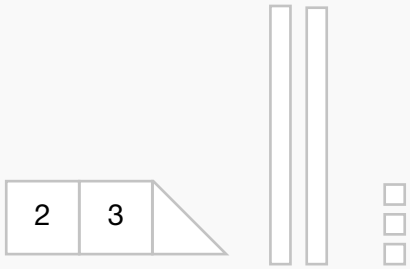
Teaching activity

'Today we are going to do some multiplying using the grid method. We are going to use the Base 10 equipment to help us.'

'Let's start by making a two-digit number to multiply.'

? **Can you make a two-digit number with the place value cards?**

If the child finds this difficult, spend a few minutes making some two-digit numbers. Make sure they understand that the size of the ten rod is linked to its value of ten. Make sure they can put out equipment to the value of the place value cards.



Choose one of the numbers, for example eighteen, and use that to multiply by three.

? **Can you make a sketch of the grid we need for multiplying eighteen by three?**

? **Can you make an estimate?**

You might want to suggest that eighteen is almost twenty, so three lots of twenty would be 20 + 20 + 20 = 60.

Therefore, if the actual answer isn't near sixty, something has gone wrong.

Remind the child that the rectangle must be big enough to write the numbers in, and it is only a sketch so it doesn't need to be to scale.

? **Which number is it best to partition? Why?** *(10 + 8 is ideal.)*

? **Can you write the numbers that we are going to multiply around the grid?**

? **Can you put out eighteen in the Base 10 equipment?**

Help the child to select one ten rod and eight unit cubes and place these on or by the grid.

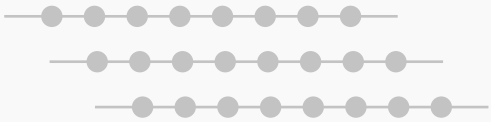
'We are going to make this eighteen three times larger.'

? **Can you make the ten rod three times bigger? What is ten multiplied by three?**

? **Why did you write thirty in that rectangle?** *(Clarify that this is three tens.)*

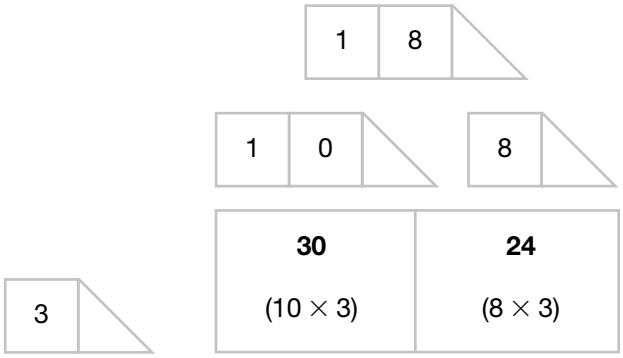
? **How can we make eight unit cubes three times bigger?**

If the child needs help with that, encourage them to count in eights three times, or make an array eight by three. Or use the model which the child tends to prefer for support, for example bead strings.



'Three lots of eight is twenty-four.'

If it will help, use place value cards to clarify:



? **Why did you write twenty-four in that rectangle?**

Help the child to use a wide range of the key vocabulary above.

? **Which numbers do we need to recombine to find the answer to eighteen multiplied by three?**

Support the child using place value cards if necessary, to recombine 30 and 24.

? **So what is the answer to eighteen multiplied by three?** *(54)*

? **Is that close to our estimate? So we are probably right!**

? **How could we record eighteen multiplied by three using just one line?**

Support the child to write:

> Eighteen multiplied by three is fifty-four
> $18 \times 3 = 54$

If they understand the partitioning well, you could write:

'18×3 can be written as (10×3) add (8×3).'

This shows how the two separate multiplications came from the place value cards and/or the equipment.

(*Note:* Remember to keep a note of some of the errors which the child is making.)

? **Do the multiplication twenty-seven multiplied by three using the grid method, but partition the numbers in three different ways. Will you always get the same answer?**

? **When you get your answers from the multiplications in the grid, why do you add the numbers together?**

Spotlight 4

Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34, or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: making decisions

Using brackets

Time 10–15 minutes

Resources

- Squared paper
- Cubes
- Place value (arrow) cards
- Base 10 equipment
- Bead strings
- Number cards (Resource sheets 1, 2 and 3)
- A note of errors which the child makes in class

Key vocabulary

partition	multiplied by
split	altogether
recombine	array
add together	row
tens	column
ones	equal groups of

Teaching activity

(*Note:* Remember you will need actual examples of errors which this child is making for this and/or the next session.)

Put out a few number cards, for example 16, 27, 24, 35, and some lower numbers to multiply by, maybe 3, 4 and 5.

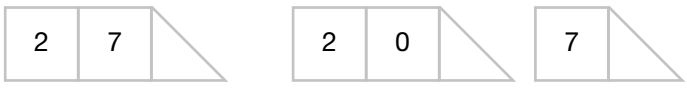
'I want you to choose some numbers that we can multiply together using the grid method.'

Help the child to select appropriate numbers, for example twenty-seven and four.

> If the child is having trouble with recombining after multiplying, you might want to start with lower numbers, perhaps twenty-one multiplied by three.

? **Which number shall we partition? Why?**

> If they need help with the partitioning, you might need to spend some time on that in another session. Do the partitioning and move on.



? **Can you sketch a grid for twenty-seven multiplied by four?**

Show the child how to write the partitioning using brackets.

'What we did was partition twenty-seven into twenty and seven and then multiply by four.'

'We can write that like this:

$27 \times 4 = (20 \times 4) + (7 \times 4)$

If the child does not understand, demonstrate clearly, maybe marking those multiplications on the grid.

	20	7
4	(20×4)	(7×4)

You might want to show these numbers with some kind of equipment that the child seems to prefer.

This is twenty-seven:

Twenty-seven made four times bigger is:

$20 \times 4 = 80$ $7 \times 4 = 28$

? **What do we need to do now with the eighty and the twenty-eight?**

? **Show me how you can recombine the eighty and the twenty-eight.**

? **When we were partitioning the twenty-seven, why did we write it as twenty add seven?**

Support the child in talking about the values of the tens and the ones.

Demonstrate how the brackets help us to find the answer.

27×4
$= (20 + 7) \times 4$
$= (20 \times 4) + (7 \times 4)$
$= 80 + 28$
$= 108$

Try some more calculations using brackets, for example:

$17 \times 5 = (10 + 7) \times 5 = \ldots$
$34 \times 6 = (30 + 4) \times 6 = \ldots$

(*Note:* The following text might relate to some of the errors that the child is making in their classwork. If that is the case, you might want to use this text. If the child isn't making these particular errors, you might want to move on to Spotlight 5, or look at specific errors which the child is making. It is important to address the errors that the child actually makes, not just suggestions of errors they *might* make!)

This text addresses one common place value error:

23 partitions into 2 and 3, so 23 × 4 is (2 × 4) added to (3 × 4).

Help the child to work this out as far as 8 + 12.

? **Can you draw a grid for 23 × 4?**

? **What is the answer to 23 × 4? Is it 8 + 12?**

? **Where did we go wrong with partitioning 23 into 2 and 3?**

If you need to do another example showing place value errors, you could go back to 27 × 4 as above.

	20	7
4	(20 × 4)	(7 × 4)

? **Why did we partition twenty-seven into twenty and seven? Why couldn't we partition twenty-seven into two and seven?**

	2	7
4	(2 × 4) = 8	(7 × 4) = 28

So twenty-seven multiplied by four is eight add twenty-eight, which makes thirty-six.

Support the child to see that 36 is *not* a good estimate for 27 × 4.

This is one way of making an estimate of the answer: 'twenty-seven is close to twenty-five, so twenty-five multiplied by four will be close to twenty-seven multiplied by four.

25 × 4 = 100 and an estimate for 27 × 4 must be close to 25 × 4, so it must be close to 100.'

? **Can you explain to me some of the things that we have to do very carefully when we are multiplying using this grid method?**

? **Look at this page in your maths book and find something you did wrong.**

? **Why do you think that was wrong?**

? **Is there anything you think you need more help with?**

Spotlight 5: a learning check

Does not apply partitioning and recombining when multiplying, for example 14 × 3 is calculated as (10 × 3) + 4 = 34, or 14 × 3 = 312, confusing the value of two-digit numbers

Opportunity for: communicating in symbols and words

Right or wrong?

Time 10–15 minutes

Resources

- Some prepared multiplications (relate these to errors which the child has made in class)
- Cubes
- Place value (arrow) cards

Check: does the child use key vocabulary?

partition	multiplied by
split	altogether
recombine	array
add together	row
tens	column
ones	equal groups of

Teaching activity

(*Note:* Remember to refer to errors that the child has been making in class.)

'This grid method for doing multiplications is very useful, because it helps us to be able to multiply two-digit numbers, which would be difficult to do in our head. But we must be careful to do the grid method without making a mistake. I'm going to show you some multiplications done with a grid and some of them are right, but some are wrong.'

You could start with some correct grids but with some numbers missing.

14 × 6

×	10	4
6		(4 × 6)

34 × 4

×	30	4
4	(30 ×)	(× 4)

? **Can you tell me how you know you are right?**

If the child needs help, let them choose another way to work the grid out, perhaps with number lines or arrays of cubes.

? **Can you explain to me how you would draw a grid to work out thirty-seven multiplied by five?**

? **Can you talk me through how you would work it out?**

? **How can you check that you are right? What would be an approximate calculation?** *(Perhaps smaller than 40 × 5 but larger than 30 × 5.)*

You might want to focus on some of the specific errors which the child is making in class.

It is probably only relevant to work through the following text if the child you are working with is making this type of error.

'You didn't always partition the two-digit number correctly.'

'Is this right?'

18 × 3 is
1 × 3 added to 8 × 3
3 + 24
So 18 × 3 is 27.

Encourage the child to make an estimate in their head of 18 × 3. 'So our answer needs to be almost 60, not 27.'

? Is this grid right or wrong for 15 × 4?

×	1	5
4	(1 × 4) = 4	5 × 4 = 20

So 15 × 4 is 4 + 20 which is 24.

Encourage the child to estimate what four multiplied by fifteen might be.

'Let's make some grids with numbers missing *(and/or errors, whichever the child needs to focus on)* to take back to class, and we will ask other children if they can solve our missing number puzzles.'

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- draw an appropriate grid to multiply a two-digit number by a one-digit number;
- partition two-digit numbers appropriately and place these on the grid;
- explain which numbers are recombined;
- check a multiplication by making an estimate;
- identify errors and be aware of their own errors;
- start to use brackets to show how the numbers are partitioned and then multiplied, then recombined.