Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: reasoning about numbers

Resources

- Place value (arrow) cards
- Abacus
- Squared paper
- Calculator

Key vocabulary

multiply by	ten times larger/smaller
divide by	two-digit numbers
digits move to the left/right	three-digit numbers

Teaching activity

Time 15–20 minutes

'We are going to look at multiplying and dividing by ten, a hundred and a thousand, and we are going to use a calculator so that we can see some patterns.'

Put a single-digit number, for example 5, into the calculator and show it to the child.

? **Can you tell me how you could change the screen to 50 in one move using the multiplication key?**

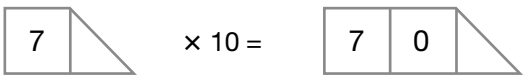
? **Can you explain what you did?**

Make sure the child uses the correct vocabulary.

> If the child is unsure, show how to key in [×] [1] [0] [=] to show 50, and use the correct vocabulary to explain it.
>
> 'I had five and I multiplied by ten and that makes fifty. Fifty is ten times larger than five because we multiplied it by ten.'

? **Can you predict what will happen if we key in seven multiplied by ten on the calculator?**

Show the numbers with place value cards.



Ensure that the child uses appropriate language to explain the process with seven. For example, 'seven multiplied by ten is seventy ' and 'seventy is ten times larger than seven'.

Go back to 5 on the calculator.

? **What happens when we multiply a single-digit number by a hundred?**

Move on to multiplying by a thousand, and to multiplying two-digit numbers by ten, a hundred and a thousand, continuing to ask the child to explain their choices of calculator buttons using appropriate vocabulary.

Ask the child to talk you through five multiplied by a hundred with place value cards, and maybe with an abacus.

If the child tends to use language such as 'you add a nought', you will need to introduce a more accurate way of explaining the changes in the numbers. For example, 'The five moves to the left and becomes ten times larger, so we put a zero in the empty column to fill the empty space.'

It can be helpful to draw up some charts on squared paper and to name the tops of the columns: units or ones, tens, hundreds, thousands, and so on.

Show how a number changes, using the calculator, place value cards or abacus, or whatever the child chooses to use.

Th	H	T	U
			5
		5	0
	5	0	0
5	0	0	0

× 10 = 50
× 10 = 500
× 10 = 5000

? **Is this true: 5 × 1000 = 50 × 100?**

? **How do you know?**

? **Is this true: 125 × 20 = 12 500?**

? **How do you know?**

Support the child to move towards explaining:

125 × 20 is 125 × 10 × 2 = 1250 × 2 = 2500

Recognising and naming larger numbers needs to be secure to understand and communicate this aspect, so you might need to spend time using the calculator to explore hundreds and thousands, encouraging the use of appropriate vocabulary to explain.

? **Can you key in sixty-five and tell me what number is ten times bigger? What do you need to do to key in ten times bigger? Now multiply by ten and see if you were right.**

? **What if I put twenty-six into the calculator – can you change it to two thousand six hundred with one multiplication? How would you write that down?**

Record some of the patterns emerging (squared paper can help).

Multiplying 48 by ten

HTh	TTh	Th	H	T	U
				4	8
			4	8	0
		4	8	0	0
	4	8	0	0	0

× 10 = 480
× 10 = 4 800
× 10 = 48 000

Do you know the names of the columns?

U = units or ones
T = tens
H =
Th =
TTh =

? **Tell me about a pattern you can see.**

? **Can you name this number: 480 000?** *(four hundred and eighty thousand)*

Help the child to talk about what they can see happening and help them by recording some generalisations in their own words. For example:

Every time we multiply by ten, the digits move one place to the left.
If we multiply a single-digit number by a thousand (that has three zeros) the single-digit number moves three places to the left into the thousands column and the hundreds, tens and ones columns all have zeros to fill the empty spaces.

? **What do you think will happen when we divide by ten?**

? **What if I key in three hundred and sixty – what single division will get the screen to thirty-six?**

Let the child explore using a calculator, and record some of their findings, explaining the process which they are using and any patterns they notice.

Dividing 960 000 by ten

HTh	TTh	Th	H	T	U
9	6	0	0	0	0

÷ 10 = 96 000

? **What shall we write down so that we remember it for next time?**

Spotlight 1

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: exploring multiplication patterns

Factor trees

Time 15–20 minutes

Resources

- Squared paper
- Calculator
- *Multiplication grid 1 or 2* (Resource sheet 35 or 36)

Key vocabulary

multiply by
divide by
digits move to the left/right
ten times larger/smaller
two-digit numbers
three-digit numbers
factors

Teaching activity

'We are going to find some factors of numbers to make it easier to multiply.'

? **Is this correct: 8 × 20 = 800?**

? **How do you know?**

Support the child to move towards:

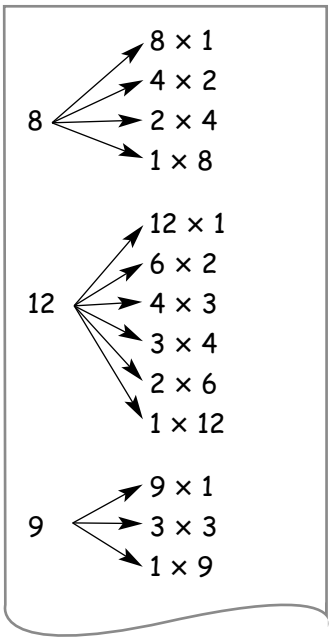
8 × 20 = 8 × 10 × 2 = 80 × 2 or 8 × 2 × 10 = 16 × 10.

Support the child to see that you can use factors of 20 to help you to multiply by 20.

Draw a few factor trees, emphasising that one, eight, two and four are factors of eight, enlisting the child's help in identifying factor pairs, using the multiplication grid for support.

> If the child finds that difficult, use cubes to make arrays. For example, with eight cubes we can make an array of one row of eight, or two rows of four.

Keep these recordings for later use.



'Some numbers have many more factors than others.'

Record some multiplication calculations using factors. Keep these recordings for use in later sessions.

? **Use factors to help you to work out eighteen multiplied by twelve.**

Let the child use a calculator to work out several examples, so that they have experience of using different factors. Encourage them to talk you through what they are doing as they do it, and to comment on anything they notice.

They could break down eighteen multiplied by twelve into factors of eighteen as well as twelve, resulting in: 2 × 9 × 6 × 2.

Whichever order you multiplied these numbers in, you would still get the same answer.

? **Tell me what we have been doing to try to make numbers easier to multiply.**

? **Give me an example.**

? **Why does using factors help?**

? **Make a factor tree for twenty-four. Now multiply twenty-seven by twenty-four using factors.**

If no one uses the nearness of twenty-seven to twenty-five to help them in this calculation, point out that they could use this as a check.

Spotlight 2

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: discussing and making decisions

Oops! banana slips

Time 15–20 minutes

Resources

- Cubes or counters

Key vocabulary

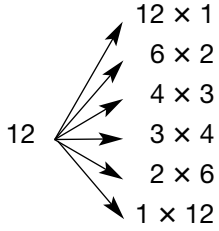
multiply by
divide by
digits move to the left/right
ten times larger/smaller
two-digit numbers
three-digit numbers
partition into tens and ones
array
factors

Teaching activity

'We are going to do some more work with factors to help us to multiply. There is more than one way of multiplying, and we need to be very clear about what we are doing so we don't slip on a banana skin!'

? **Tell me the factors of twelve and draw a twelve factor tree.**

Support the child to draw:

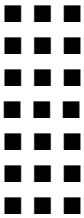


'We're going to work out seven multiplied by twelve using these cubes and a pair of factors of twelve. Let's start by using factors three and four and I'll record for you what we do.'

7 × 12 can be written as 7 × 3 × 4.

? **Make an array for seven multiplied by three with cubes.**

This array shows 7 × 3.

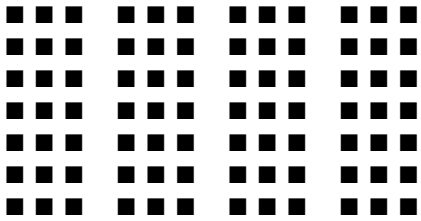


? **Look carefully at 7 × 3 × 4 and tell me what we need to do to our array of 7 × 3 to make it show 7 × 12.**

Support the child, moving towards the idea that we need four lots of the 7 × 3 cubes to show 7 × 12.

This array shows four lots of 7 × 3 cubes:

7 × 3 × 4



7 × 3 = 21 and 21 × 4 = 84.

? **Rearrange these same cubes to give another way to show 7 × 12.**

The child might need support here to recognise that they need to choose another pair of factors of twelve, for example six and two, then arrange the same eighty-four cubes into different arrays, for example an array of seven multiplied by six twice.

? **What did you learn today about the way factors can help us to multiply?**

? **How would you explain to someone why multiplying eighteen by twelve gives the same answer as 2 × 9 × 2 × 6?**

Spotlight 3

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: exploring place value and multiplication patterns

Multiply it

Time 15–20 minutes

Resources

- Place value (arrow) cards
- Wipe-clean place value board
- Calculator

Key vocabulary

multiply by	ten times larger/smaller
divide by	two-digit numbers
digits move to the left/right	three-digit numbers

Teaching activity

'We're going to do some more multiplying by ten and a hundred today. Then we will multiply by twenty and two hundred using factors.'

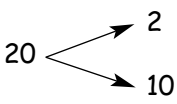
? **What can you remember about multiplying by ten and a hundred?**

? **What is three multiplied by ten and three multiplied by a hundred?**

If the child has forgotten, go over multiplying by ten and a hundred using a calculator and place value cards, recording on a place value board. Try to work out which model the child seems to prefer.

Record some multiply by ten and multiply by a hundred calculations. Then extend these to some related multiply by twenty and multiply by two hundred calculations, guiding the child towards the patterns using appropriate vocabulary to support description of the process.

3 × 10 = 30 and 3 × 20 = 60 as this is 3 × 10 x 2
because factors of twenty are ten and two.



3 × 100 = 300 and 3 × 200 = 600 as this is 3 × 100 × 2.

If the child needs it, go through this pattern again with four and five. Use a calculator so that the child can focus on the patterns, rather than worrying about getting the calculation right. Record the patterns of numbers. Encourage the child to explain in their own words what they notice.

Again, a place value board and/or place value cards might assist in building up appropriate images.

5 × 10 = 50 and 5 × 20 = 100 as this is 5 × 10 × 2

5 × 100 = 500 and 5 × 200 = 1000 as this is 5 × 100 × 2

? **Tell me how to use the ten times table to multiply six by eighty.**

Support the child in finding a way of doing this, for example 6 × 10 × 8.

Encourage the child to work out all the versions they have managed to identify for six multiplied by eighty.

Repeat with another example.

? **What shall we write down today that is important to remember for next time?**

Spotlight 4

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: exploring patterns with division

Division patterns

Time 15–20 minutes

Resources

- Place value (arrow) cards
- Wipe-clean place value board
- Number cards
- Squared paper
- Calculator

Key vocabulary

multiply by
divide by
digits move to the left/right
ten times larger/smaller
two-digit numbers
three-digit numbers

Teaching activity

'We are going to make some division patterns today, dividing by one, by ten and by a hundred.'

? **Can you show me the division key on your calculator?**

? **What can you tell me about these number patterns?**

3 ÷ 1 = 3

30 ÷ 10 = 3

300 ÷ 100 = 3

? **Can you make another pattern with three calculations involving division by one, ten and a hundred, all with the answer 4 on your calculator?**

? **Explain your choice of numbers.**

Build on what the child does.

If the child needs a bit more guidance, you could start off a pattern and, if they still seem not to be following, use place value cards or a place value board and number/arrow cards, or whatever the child seems to prefer.

Th	H	T	U
3	0	0	0
	3	0	0

÷10 = 300
÷10 = 30
÷10
÷10

? **Tell me about your pattern.**

Encourage the child to make generalised statements such as:

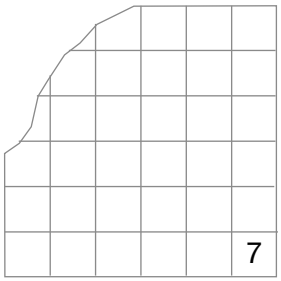
'When you divide by ten the digits move one place to the right.'

? **What do you think would happen if we divided by a hundred?**

If the child can't see that the digits would move two places to the right each time, explore this on a calculator with them, using any supporting equipment which the child seems to prefer to use. Encourage the use of appropriate vocabulary.

Move on to using another squared paper grid.

'Show me a division by ten pattern that ends in seven.' (Ending in forty-seven would be more challenging.)



? **Was there anything we worked on today that you thought was new knowledge for you?**

? **I divided my mystery number by a hundred and did that division by a hundred four times and ended with eight. What was my mystery number?** *(Eight hundred million, 800 000 000)*

(*Note:* The division is by one hundred four times, not by four hundred! You could use this as a moment to point out that we have to be very careful about the language we use in mathematics.)

Spotlight 5: a learning check

Misuses half-understood rules about multiplying and dividing by powers of ten and the associative law, for example 145 × 30 = 145 000

Opportunity for: discussing and explaining

Beat the calculator

Time 5–15 minutes

Resources

- Wipe-clean place value board
- Calculators
- A group of four or more children (a whole class is ideal)

Check: does the child use key vocabulary?

multiply by
divide by
digits move to the left/right
ten times larger/smaller
two-digit numbers
three-digit numbers
factors

Teaching activity

'Today we are going to play a game to help you practise multiplying and dividing by multiples of ten.'

This game, **Beat the calculator**, is a small- or large-group game and works well with the whole class. The game can be played at many different levels and can be used to give practice and consolidation of a wide range of learning objectives.

How to play

1. Divide the children into two groups. Give the children in one group a calculator each but give nothing to the second group. Tell the second group that they can make jottings on paper if they want to.
2. Read out a series of calculations. Here are a few examples:

   45 × 100 100 × 2 30 ÷ 10 29 × 100 450 ÷ 10

3. The group with calculators must key each calculation in, and not call anything out until they see the answer on their screen.
4. The group with no calculators must work out the calculation in their heads, or with brief jottings, and then call out the answer.

   (Depending on the calculations which you read out, the group without the calculators could beat the children with calculators! That is a very valuable lesson, making this game one to play at least once a term.)

? Why could you work that out so quickly in your head? Can you tell us how you did it? Did anyone do it a different way?

5. Give a hundred points to the group that shouts out the answer first and keep the scores on the board.
6. After about five or ten goes, swap the groups round and call out five or ten more calculations.

Variations

- Let one of the children work out about ten questions of their own which they think could be worked out more quickly mentally than on the calculator. That child is then the leader and calls out their questions.

? Can you explain to me how you worked that out? Is there another way to do it?

- This can be a good game to share with parents, especially those who think that calculators harm children's mathematics!

Learning outcomes

By the end of this set of activities children should be able to:

- use and understand connected mathematical vocabulary;
- tackle related learning tasks with increased motivation and confidence;
- multiply and divide by ten and a hundred;
- describe what is happening to the digits in multiplying and dividing by powers of ten;
- multiply by multiples of ten, a hundred and a thousand;
- use factors to support multiplication;
- use a calculator appropriately.