Discards the remainder; does not understand its significance

Opportunity for: understanding and communicating mathematical ideas and symbols

Resources

- Cubes
- Number lines
- Paper plates and pretend sandwiches

Key vocabulary

divided by
divided into
divided between
shared between
left over
remainder
hops of

Teaching activity

Time 10–15 minutes

'We are going to do some division today with these sandwiches, and we are going to think carefully about what we do when we have some items left over, a remainder.'

? **Can you tell me what you think having a remainder after doing division means?**

Build on what the child says.

? **Can you show me what you mean if we had eight sandwiches and we had to put them three to a plate for a party?**

Support the child putting three sandwiches on each plate, saying 'eight divided by three'.

? **Can you show me how you would write down eight divided by three?**

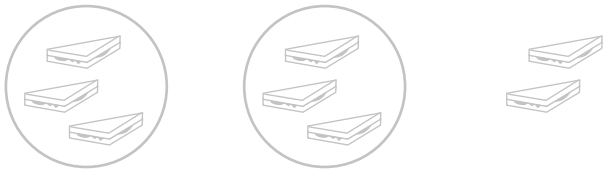
If the child doesn't know, help them to write 8 ÷ 3 as you say 'eight divided by three'. You will need to give them a few more examples to write down, such as:

seven divided by three;

sixteen divided by five;

nineteen divided by two.

Make sure that they can write these in symbols and read them back to you in words. Aim for 'divided by' as the main words you use, as the example of the sandwiches was a grouping model.



Come back to the calculation eight divided by three and talk through the sandwiches on the plates. Try to end up with a calculation.

8 ÷ 3 = 2 complete plates of three sandwiches with two sandwiches left over,

or 8 ÷ 3 = 2 and a remainder of two sandwiches.

You might want to show children this recording:

8 ÷ 3 = 2 remainder 2

? **If we have eight sandwiches and each plate has to have three sandwiches, how many complete plates can we make? So how many people at the party can have a plate of three sandwiches?**

? **What about these two sandwiches left over? What could we do with them?**

Make a note of responses. (You will be adding to this list throughout this set of activities.)
We could:

- eat them;
- divide them up into smaller bits and share them around...

Clarify that you probably wouldn't just leave the sandwiches to go hard and stale. You would do something with them.

? **Can you choose something on the table to show me how you would divide seventeen muffins by three?**

? **How many muffins will you put in each group if they were 'divided by three'?**

Build on where the child is with their thinking and record:

Seventeen divided by three.

Seventeen muffins put into groups of three.

Seventeen divided by three is five groups of three and two muffins left over.

Five people can have three muffins each.

? **What might we do with the two muffins left over?**

If the child needs more help, give some more practical examples.

'What if we had sixteen sandwiches for the party and we wanted to give people five sandwiches each. Let's write the division calculation sixteen divided by five.'

$16 \div 5 =$

Let the child work this out any way they want (cubes, number lines or moving sandwiches onto plates) and observe carefully what they do. If they share out with a 'one for you, one for me' method, let them continue with this but make a note that you want to move them on from it during this work on division.

If they share they might end up with five groups of three and one left over, because they were using a dividing between five model rather than dividing by five.

You could build on that and ask them what they will do with that one left-over sandwich.

Add their ideas to your list of their suggestions of practical things they could do with the remainders based on the context.

'Let's put out some plates and put five sandwiches on each plate, that is sixteen divided by five.'

? **How many fives do you think there are in sixteen?**

Try to get to the response, 'There are three fives in sixteen and one left over.'

(You can support a child with a number line to work this out.)

'Look at our list of things we thought we could do with left-over sandwiches. Mostly we thought that we wouldn't just leave the sandwiches to go hard and mouldy! We can't just ignore the left-over bits after a division!'

? **What did you learn today?**

Spotlight 1

Discards the remainder; does not understand its significance

Opportunity for: solving problems in a real-life context

Tom's tickets

Time 15–20 minutes

Resources

- Cubes
- Number lines
- Money

Key vocabulary

divided by
divided into
divided between
shared between
left over
remainder
hops of

Teaching activity

grouping model

'We are going to do some more about remainders today, and this time we are going to work with money.'

? **If Tom had £15 and tickets for the disco were £6 each, how many tickets could he buy?**

Let the child work this out any way they want and observe carefully what they do.

'Tell me what you are doing as you do it.'

Support the child using the key vocabulary.

? **What division calculation would you write for finding out how many £6 tickets Tom could buy?**

If the child doesn't know, help them to record in some way that makes sense to them.

A number line can show remainders clearly.

'We need to hop back from 15 in steps of six.'

Two hops of £6

'We have £3 left over.'

? **If we weren't thinking about pounds and tickets, how would you work out fifteen divided by six?**

Try to end up with 15 ÷ 6 = 2 remainder 3.

'So Tom can buy two tickets and he has £3 left over.'

? **If you were Tom, what would you do with that £3 left over?**

Record the responses, such as:

- Save the money for something else.
- Ask Dad for £3 so that another ticket can be bought.
- Give the money to a friend so that they have enough to buy a ticket for themselves.

? **At the disco, cold drinks are eighty pence each, so how many can Tom buy for £3?**

grouping model

? **Can you make an estimate first? About how many drinks? Just a few, or more than five?**

Encourage the child to see that not many drinks can be bought, because 80p is quite close to £1.

Let the child work out the exact answer any way they want and build on what they say and do.

If the child doesn't know what to do, counting back in groups of 80p might help.

? **How much money does Tom have left over?**

? **What might he do with that money?**

(*Note:* You might want to note how familiar the child seems with money. For some children, putting a task in terms of money can help enormously because they are used to handling money. However, some children have had so little experience with shopping that money tasks still confuse them. They will need to do further problem solving with money and play money games. See Spotlight 5.)

? **Can you tell me what you have learned about remainders today?**

Explain that you are going to keep their list of ideas for what they could do with remainders and add to it next time.

Spotlight 2

Discards the remainder; does not understand its significance

Opportunity for: reasoning about numbers

Let's go to the beach!

Time 15–20 minutes

Resources

- Cubes
- Base 10 rods
- Number lines
- Calculator

Key vocabulary

divided by
divided into
divided between
shared between
left over
remainder
hops of

Teaching activity

'Today you are going to pretend to be the after-school club leader, and you are planning a day out to the seaside in half-term for every child in the school. There are two hundred and forty children and we don't want to leave any of them behind.'

'Each coach can take fifty children. (There is also space for two adults because there are fifty-two seats, but leave that out of the calculation.)'

? **How many coaches do you need to take all two hundred and forty children to the beach?**

Let the child work this out in any way they want. Establish that the calculation is written down as: 240 ÷ 50 =

? **How many fifties are there in two hundred and forty?**
Or how many groups of fifty can we make from two hundred and forty?

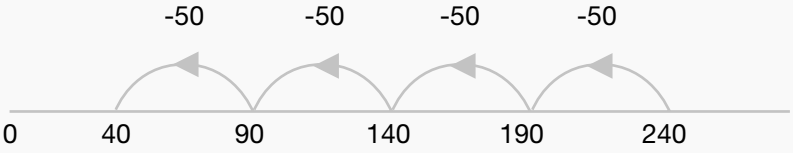
? **Can you estimate this first?**

If they can't, let them move on so you can see what they are thinking. If they look as if they are going to count out two hundred and forty cubes, you need to stop them and ask them if they can find a quicker way of doing it.

Two hundred and forty as twenty-four Base 10 rods would be better. These could be put into groups of fifty (five 10-rods).

Using a number line is a good method. They can use a calculator to check their numbers.

Count back in steps of fifty



? **How many steps of fifty?**

Alternatively, let them use a calculator from the start. It can lead them right into the trap you almost expect them to fall into!

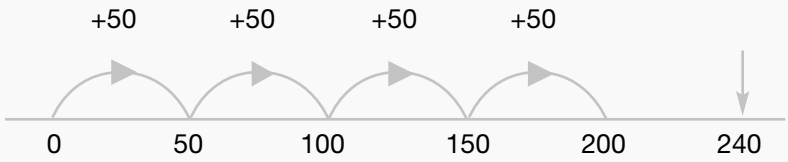
On the calculator 240 ÷ 50 = 4.8, so if they tell you they need 4.8 coaches, this is the moment to ask them just what 4.8 coaches would look like!

Worked out on a number line, counting back from 240 in steps of fifty, there are four lots of fifty to get to 40 on the number line, then the left-over forty children need a coach as well.

If the child is confused about the larger numbers, you could do an addition calculation, maybe using 10-rods of some kind to support the large numbers.

'50 + 50 + 50 + 50 = 200. That is four full coaches.'

You can show this by counting up on the number line.



'Then there are another forty children.'

? **What could you do if you were organising the trip and there were only forty children to put on the last coach? You won't just ignore them, will you? They don't want to be left behind!**

? **What could you do with the extra forty children?**

Record some of the child's responses to add to the list of responses from earlier work.

Maybe there is...?

Or maybe...?

Remind the child of the list of their ideas of things to do with remainders and add to it their responses from today.

? **Tell me what you learned today about remainders.**

Spotlight 3

Discards the remainder; does not understand its significance

Opportunity for: making a connection between mathematical symbols and real-life contexts

Number line remainders

Time 10–20 minutes

Resources

- Number lines

Key vocabulary

divided by	left over
divided into	remainder
divided between	hops of
shared between	count back

Teaching activity

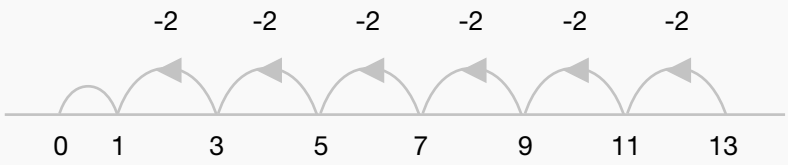
'We are going to do some more division calculations that have a remainder, and we are going to see if we can make up some stories to go with the calculations.'

? **Can you write down thirteen divided by two?**

? **How can you work that out on a number line?**

Support the child writing 13 ÷ 2 = and saying 'thirteen divided by two'. If they are unsure about drawing a number line for themselves, they can use a blank commercial line, but you might need to start them off by marking the 13 at the right-hand end.

Six hops of two and one left over



13 ÷ 2 = 6 remainder 1

There are six twos in thirteen with one left over.

'Let's try to make a story to go with thirteen divided by two.'

If the child can't think of a story, you could give them a starting point. For example, 'Dan has thirteen Easter eggs and his mum says he mustn't eat more than two a day.'

? **For how many days can he eat two Easter eggs?** *(Six with one egg left.)*

Build on the child's own ideas where you can.

Apples, biscuits, sweets, muffins, pens and £1 coins could also be used to help the child to show how they know their story works.

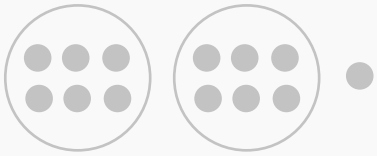
? **What could Dan do with the left-over Easter egg?**

Develop the child's ideas for how to deal with remainders and add these to their earlier list of ideas.

It is likely that some children might say, for thirteen divided by two (which is a grouping story), 'Thirteen sweets shared between two children is six each and one left over', making it a sharing story that does not fit with the number line jumps of two above.

grouping model

The image that might fit with thirteen divided *between* two is:



sharing model

These children need more experience with using the words 'divided by two' and 'how many twos make thirteen' and with using the image of a number line, counting back in a known step size (two in our example), rather than reading the division sign always as 'sharing between'.

You can give some further examples of calculations that can be solved on a number line, focusing on 'how many steps back'.

? **How could you solve the problem seventeen divided by two on a number line?**

? **Can you make a number line drawing to show how many fours there are in twenty-one?**

Spotlight 4

Discards the remainder; does not understand its significance

Opportunity for: solving practical problems and making decisions

Two and a bit

Time 10–20 minutes

Resources

- Cubes
- Number lines
- Money
- Apples, biscuits, or soft modelling material

Key vocabulary

divided by
divided into
shared between
left over
remainder
hops of
count back

Teaching activity

'You are getting good at remainders and today we are going to find some more remainders.' Look together at the child's list of suggestions for dealing with left-over items.

? **Supposing you and your friend had been washing cars to earn some money and you earned £2.55. How could you divide that equally between the two of you?**

Support the child, building on what they say and using coins (or pencil and paper jottings if the child chooses these) to clarify the division.

If the child cannot divide £2.55, you will need to spend time with coins, dividing money amounts such as £2, £3, £2.50, and so on, making sure they can recognise coins, can name them, and know how to both double and halve a simple amount of money.

Divide the £2.55 into two lots of £1.27 and show the 1p left over. Remind the child of odd and even numbers.

? **What might the friends do with the 1p?**

If the child needs more work with money, you could make up some more practical money divisions, such as £2.55 divided between three friends, and so on.

? **Can you work out how a packet of eight sausages could be divided between a family of five people?**

Talk the child through what some of the different outcomes could be. Encourage them to use modelling material or objects to look at the possibilities.

This could be a time to let the child record in their own way, perhaps with pictures that can be taken back to class and talked through with the rest of the class.

> Eight divided between four would be two sausages each, but divided between five is less than two each.

If the child is having difficulties with this, you will need to do further practical examples with objects, until their language has developed and they can divide small numbers reasonably confidently, making a sensible decision about what can be done with different remainders.

Clarify with the child that remainders are important and we need to think about them carefully and decide what is best to do.

'We can break a biscuit into halves, but we can't tear a £10 note in half! We have to consider remainders carefully to fit with what we are dividing.'

? **What shall we record from today that is important to remember?**

Spotlight 5: a learning check

Discards the remainder; does not understand its significance

Opportunity for: discussing, explaining and reasoning about numbers

Hand over the beans

Time 5–15 minutes

Resources

- Cubes
- Number lines
- Dried beans
- Bag to contain number cards
- Number cards 1 to 50 (Resource sheets 1, 2, 3 and 4)
- ÷ and = cards (Resource sheet 8)
- *Multiplication grid* (for example, Resource sheet 35)
- At least two players

Check: does the child use key vocabulary?

divided by
divided into
divided between
shared between
left over
remainder
hops of

Teaching activity

'We're going to play a game today called **Hand over the beans** to practise what we've learned about remainders.'

Use number cards to suit your children. You could have 12, 14, 15, 16, 18, 20, 21 in a bag and 2, 3, 4, 5, 6, 7, 8, 9, 10 on the table.

Some children might need to play the second variation of this game first if their knowledge of linking divisions to multiplication tables is not secure.

Put the bag with the larger numbers on the table, so the cards can't be seen, and put the 2, 3, 4, 5, 6, 7, 8, 9, 10 cards face up on the table and spread out (see variation on page 12).

How to play

1. Everyone starts with five beans. Someone will be bean banker and look after the rest of them.
2. Player 1 takes a card out of the bag, for example 15. They must select a smaller number from the table so that they can make a division calculation that does have a remainder. They can use cubes or number lines to work it out.
3. Player 1 must read and write their calculation and explain why they think they are right. If everyone agrees that they are right, they will win as many beans as there are in their remainder.
4. If the other players think player 1 is wrong in what they say or write, the other players can challenge them and must prove why player 1 is wrong. The other players can draw a number line, or look at the multiplication grid. If they do prove that player 1 is wrong, player 1 must give a bean to each of the other players!
5. The other players then take their turn to take a card from the bag and choose a smaller number from the face-up cards so that they make a division with a remainder.
6. The winner is the player with the most beans at the end.

? **Can you explain to me why you chose that card to go with 21?**

? **How do you know you are right?**

Variation

- Once the players have understood the game, help them to see that if they select their smaller number carefully, they can end up with larger remainders. For example, 15 ÷ 9 = 1 remainder 6, giving a much larger number of beans to win than with 15 ÷ 2 = 7 remainder 1.

? **Could you have chosen another card to make your remainder larger?**

? **Could you change the number cards you play with so that you can make even bigger remainders?**

- Put all the cards face down in two groups or piles. Take a card from each pile as the numbers to work with. (This is more reliant on luck but can also be quite a bit harder.)

This variation is an appropriate problem-solving task for a whole class.

- Play so that you don't have a remainder. If 18 is taken out of the bag, that player must choose 2 or 3 or 6 or 9 from the smaller numbers and make their division sentence, read it and explain it, showing that there is no remainder.

18 ÷ 6 = 3 and no remainder

If the player is right, they win a bean. If they are wrong, they must give a bean to each of the other players.

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand a range of strategies to deal with remainders in real-life contexts;
- use a number line to show division and remainders;
- link a division calculation to a real-life example.