Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: communicating with a range of mathematical language and symbols

Resources

- Cubes/apples/rods/counters
- Calculator
- Empty number lines
- Plastic, paper or dough fraction pieces or *Fractions* (Resource sheet 42)

Key vocabulary

remainder, left over, fraction, divided by, whole one, half, quarter

Teaching activity

Time 10–20 minutes

Explain to the child that today's activity will help them to be clearer about how to deal with remainders when they work out divisions.

? **What do you think it means to have a remainder after working out a division calculation?**

'Choose something on the table to show me what you mean.'

Listen to what the child says and make a note of what they do.

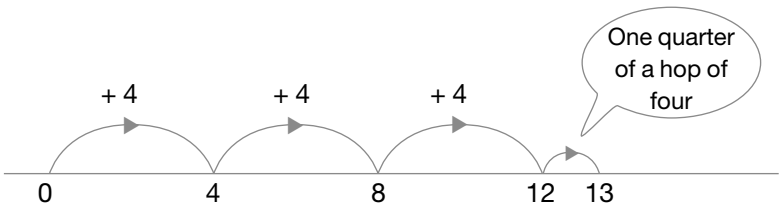
If the child seems to be unsure about remainders, give them a simple calculation to work out.

? **Can you work out thirteen divided by four? You can use any of the equipment to help you.**

It is likely that some children will work out thirteen divided by four by using a sharing method – putting thirteen counters into four groups. Remind the child that when we say 'divided *by* four' we mean putting the items into fours, not shared between four.

Let them work out how to show the calculation with cubes/dough/apples, and remind them how to use a number line to work out the calculation. (Hops backward or forward are both fine – the child should choose whichever representation they prefer.)

'There are three hops of four to get to 12 and then a quarter of a hop to get to 13.'

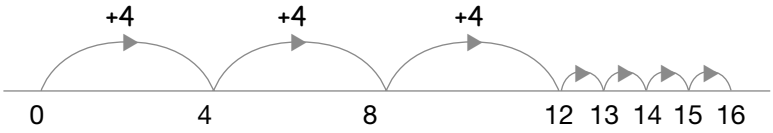


If the child seems to be unsure about naming fraction parts, you might need to spend a few moments identifying halves, quarters and three-quarters using paper or plastic fraction parts.

Record with the child, encouraging them to use a range of appropriate language and using pictures and symbols that help them. (The number line picture shows remainders as fractions in a powerful way, so focus on that.)

13 ÷ 4 = 3 remainder 1

That remainder is one quarter of the way towards 16, which is the next hop of four.



So 13 ÷ 4 = three groups of four and a quarter of a group of four.

13 ÷ 4 = 3¼ because the remainder of one is a quarter of a whole hop.

Try another example and record it with the child.

? **What is fifteen divided by two?**

? **What picture do you have in your head for that?**

Talk the child through the images, putting fifteen into equal groups of two, steps of two on a number line. Dividing by two means 'how many twos in fifteen?'

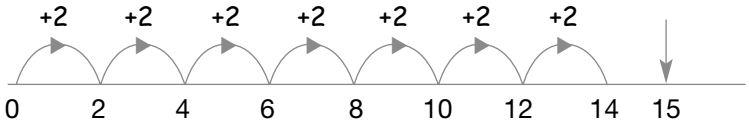
15 ÷ 2 = 7 remainder 1

You can say this as: how many twos make fifteen? It is seven with one left over.

Fifteen divided by two makes seven groups of two and one left over.

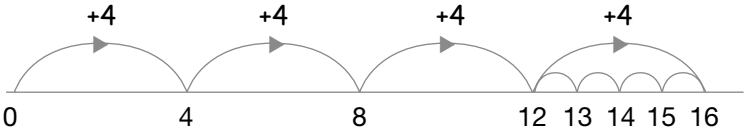
Fifteen buns put into packs of two will make seven packs and one bun left over (which is half a pack).

We can do seven hops of two and a half hop of two (one). A whole hop of two at the end would get us to 16, but we only want to go halfway (to 15).



'Let's draw thirteen divided by four on a number line and look very carefully at that last hop.'

'Thirteen divided by four means we need to take hops of four.



'We can make three complete hops of four and the remainder is a quarter of a whole hop of four.

'The whole hop of four would get us to 16, but we only want to get to 13. So thirteen divided by four is three and a quarter groups.'

? **If you key in thirteen divided by four on your calculator, what do you think the answer would be? Explain how you know that.**

If the child is unsure about how fractions look on a calculator, you might choose to draw up a table of fractions and how they relate to decimals.

The child can key these in as 1 ÷ 2, 1 ÷ 4, and so on.

1 ÷ 2 = 0.5, which is a half ($^1/_2$).

1 ÷ 4 = 0.25, which is a quarter ($^1/_4$).

1 ÷ 3 = 0.3333, which is a third ($^1/_3$).

3 ÷ 4 = 0.75, which is three-quarters ($^3/_4$).

? What shall we record today that is important to remember for next time?

Keep recordings of any pictures and symbols which the child uses, ready for later work.

Spotlight 1

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: solving real-life problems

Apples and buns

Time 15–20 minutes

Resources

- Dough and blunt knife, apples, carrots, buns
- Plastic or paper fraction parts or *Fractions* (Resource sheet 42)
- Calculator

Key vocabulary

remainder	half
left over	quarter
fraction	three-quarters
divided by	thirds
whole one	fifths

Teaching activity

'We are going to do some more division today. We are going to find answers where we interpret the remainder as a fraction.'

Using sixteen apples, or other objects, ask the following:

? **We have sixteen apples for three horses. How many apples is that each? What about the one left over?**

'Each horse can have five apples and there is one left over, so 16 ÷ 3 = 5 remainder 1.'

? **How can we divide up the one apple left over?**

Support the child to show how the apple can be cut into three equal parts.

? **What name do we call each of these parts?**

If the child can't name the fraction as a third, spend a bit of time naming fraction parts. Check that they know half, quarter, three-quarters, third and fifth.

Using nine carrots, or other objects, ask the following:

? **We have nine carrots for two rabbits. How many carrots is that each?**

9 ÷ 2 = 4 remainder 1, but if we divide the left-over carrot by two, we can write this as:

9 ÷ 2 = $4\frac{1}{2}$.

The calculator showed an answer of 4.5.

Using seventeen buns, or other objects, ask:

? **We have seventeen buns for four children. How many buns is that each?**

17 ÷ 4 = 4 remainder 1

17 ÷ 4 = $4\frac{1}{4}$

The calculator showed an answer of 4.25.

? **Look back at what we have done today and show me a division calculation where the answer includes a fraction.**

Spotlight 2

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: identifying patterns

Looking for patterns

Time 15–20 minutes

Resources

- *Multiplication grid 1 or 2* (Resource sheet 35 or 36)
- Paper or plastic fraction pieces or *Fractions* (Resource sheet 42)
- Number line
- Counters

Key vocabulary

remainder, left over, fraction, divided by, whole one, half, quarter, three-quarters

Teaching activity

'We are going to divide by four and find some remainders, and then work out those remainders as fractions.'

? **Can you remember from the last session which fractions we might make when we are dividing by four?**

Record a list of divisions, asking them:

? **What do you think comes next in the pattern?**

Use the language 'nine divided by four is two remainder one', and then explain that the remainder of one can be divided by four to make four quarters. So the answer to nine divided by four is two and a quarter.

9 ÷ 4 =
10 ÷ 4 =
11 ÷ 4 =
12 ÷ 4 =
13 ÷ 4 =
14 ÷ 4 =

? **Can you predict which of these division calculations won't have a remainder? How do you know?**

If the child is unsure about this, encourage them to refer to the four times table on their *Multiplication grid* (Resource sheet 35 or 36).

If the child is not following this, you might want to start the pattern on another bit of paper with 4 ÷ 4 = 1 with no remainder.

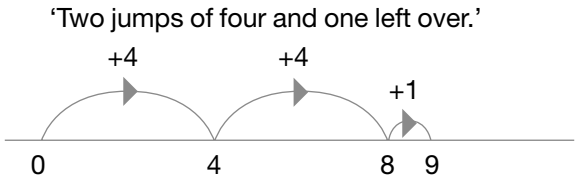
5 ÷ 4 has a remainder of 1, so 5 ÷ 4 = 1 remainder 1, which is $1\frac{1}{4}$.

6 ÷ 4 = $1\frac{2}{4}$ or $1\frac{1}{2}$

7 ÷ 4 = $1\frac{3}{4}$ and so on.

Try to establish that all the calculations that can be related to the four times table have no remainders because these numbers (4, 8, 12, 16, and so on) are multiples of four.

You might choose to model the calculations on both a number line and using counters put into fours.



'Nine divided by four is two groups of four, and one left over.'

'If we divided that one left over by four, we get a quarter.'

'Nine divided by four is two groups of four and one quarter of a group of four.'

? **What patterns can you see in the fraction parts of the answers?**

$9 \div 4 = 2$ r 1 or $2\frac{1}{4}$

$10 \div 4 = 2$ r 2 or $2\frac{2}{4}$ or $2\frac{1}{2}$

$11 \div 4 = 2$ r 3 or $2\frac{3}{4}$

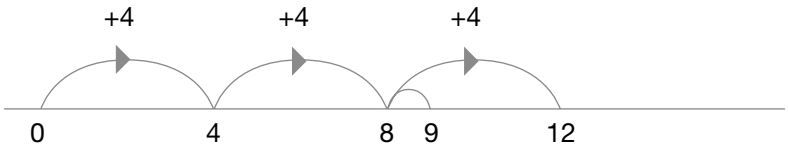
$12 \div 4 = 3$

$13 \div 4 = 3$ r 1 or $3\frac{1}{4}$

$14 \div 4 = 3$ r 2 or $3\frac{2}{4}$ or $3\frac{1}{2}$

etc.

You could show this on number lines. Each time one more quarter step is added.



'Nine divided by four is two jumps of four and one quarter jump.
Ten divided by four is two jumps of four and two quarter jumps.
Eleven divided by four is two jumps of four and three quarter jumps.
Twelve divided by four is three whole jumps of four.'

? **Predict the next calculation in the pattern.**

? **What fraction will the remainder make? How do you know?**

? **Did you learn anything new today about remainders when we work out division calculations?**

? **Write in words two division calculations that have the words 'divided by' and that have a fraction as part of the answer.**

? **Write a division calculation that has one-fifth as part of the answer.**

Spotlight 3

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: reasoning about numbers

Sensible answers

Time 15–20 minutes

Resources

- 28 eggs (or egg substitutes, such as ping pong balls)
- 5 egg boxes
- Calculator
- Money

Key vocabulary

remainder
left over
fraction
divided by
whole one
half
quarter
three-quarters
thirds
fifths
sixths

Teaching activity

'Sometimes when we are working out division calculations with real things, we have to be very careful about how we write the answer, making sure it is a sensible answer. We're going to work on this today.'

? **How could you divide twenty-eight eggs by six?**

Let the child work this out any way they want.

> *Note:* If the child is trying to share the twenty-eight into six groups, they need to be reminded that the calculation is to *divide by six*, so that means making equal groups of six.

? **How would you write down twenty-eight divided by six?**

28 ÷ 6 = 4 remainder 4

> If the child is finding it difficult, let them start with twenty-eight eggs and put them into groups of six (in egg boxes if available).
>
> Six multiplied by four is twenty-four and there will be four eggs left over, so one egg box will not be full but can hold the remainder of four eggs.

? **Do you think it might help to check it on a calculator?** *(This gives an answer of 4.6666666.)*

? **What is the sensible answer to twenty-eight eggs divided by six? How do you know?**

Establish that working out the calculation so that the answer has a fraction doesn't always give us a sensible answer.

? **Three children have been doing some odd jobs around the house, and for their neighbours, and, altogether, they have earned £13 which they want to divide equally between themselves. How much will they get each?**

If the child needs support, let them work it out using coins or jottings.

£13 divided into three equal groups can be worked out as:

3 × 4 = 12 so each child would get £4, with £1 left to divide up.

100p divided by three is 33p and a bit left over.

? **Do you think the calculator might give us a better answer?**

13 ÷ 3 = 4.33333

Again, stress that calculator answers don't always make sensible answers.

Talk about what the children might do once they all have £4.33.

? **Is it possible to divide the left-over one penny between three people?**

? **How might they solve their problem?** *(Accept any sensible suggestion – they might give the penny to the charity box in the corner shop, or they might have only £4.30 each and give the remaining ten pence to charity or to a younger child who was too small to help them.)*

? **For indoor hockey, the thirty-two children in the class need to get into teams of ten.**

? **What division do you need to do to find out the number of teams?**

Help the child to write thirty-two into teams of ten as 32 ÷ 10 = 3 teams of ten and two children left over.

'Let's see what the calculator says for thirty-two divided by ten.'

? **If you think very hard, can you predict the answer to thirty-two divided by ten?**

The calculator gives an answer of 3.2, but 0.2 of a team doesn't have much meaning!

? **How could the thirty-two children solve their problem with the two extra children?** *(Accept a wide range of suggestions.)*

? **What do you have to do to make sure you have chosen a sensible answer when you are giving answers with remainders or fractions?**

Spotlight 4

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: developing mental images

Counting backwards or forwards?

Time 10–20 minutes

Resources

- Soft modelling material

Key vocabulary

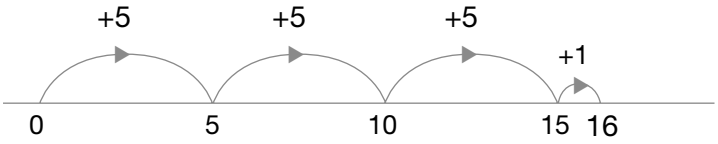
remainder	quarter
left over	three-quarters
fraction	thirds
divided by	fifths
half	

Teaching activity

'We are going to do some divisions by working them out on an empty number line.'

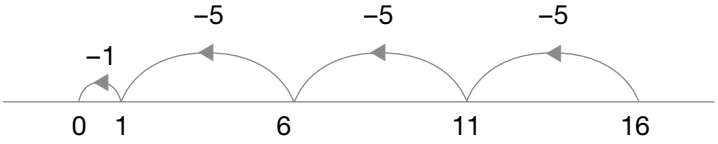
? **Can you show me how you could work out sixteen divided by five, drawing yourself an empty number line?**

If the child struggles with this, help them to draw a line and draw steps of five. They can start at 0 and count forwards in fives, or start at 16 and count back in fives. Either way they should have a remainder of one, which can be interpreted as one-fifth.



'Three steps of five and a step of one.'

'Sixteen divided by five is three groups of five and a remainder of one divided by five is one-fifth.'



? **Which way do you like to do your dividing on a number line? Do you like counting forwards, or do you prefer counting backwards?**

Try both ways. What do you notice?

16 - 5 - 5 - 5 = 1

? **What shall we do with the one left over? How will we divide it into five equal groups?**

? **What is the answer to sixteen divided by five?** *($3\frac{1}{5}$ or 3.2)*

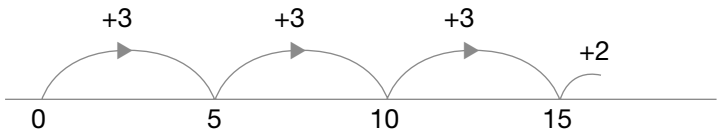
? Work out seventeen divided by three.

'Seventeen divided by three. That is five steps of three along the number line and a remainder of two.'

17 ÷ 3 = 5 remainder 2

The remainder two when divided by three gives two-thirds ($^2/_3$).

Because it is divided by three, each step is a step of three, except for the remainder, which is two bits out of three.



'I can count forwards in steps of three.'

The child might find the fraction 'two-thirds' too demanding, so you might need to fold paper into thirds to demonstrate two-thirds. (Folding a piece of A4 paper into thirds to put in an envelope is a skill that some adults find difficult!)

Ask the child to work out nineteen divided by four.

If the child is finding this too challenging, you might want to repeat this lesson with different numbers next time and try to put emphasis on doing the steps just in the direction the child seems to prefer. Many children like to start at 0 because counting forwards uses numbers that are familiar to them and they can find them on their multiplication grid.

'Let's record some of the words and ideas we use for dividing.'

Using multiplication tables helps me.

How many threes make 18?

18 ÷ 3

Eighteen divided into three equal groups.

9 ÷ 2
will have a remainder

I can hop forward on a number line to divide by a number:
5 + 5 + 5 + 1

I know all this about dividing

16 – 5 – 5 – 5 = 1

Remainders can be interpreted as fractions.

Spotlight 5: a learning check

Has difficulty, when appropriate, interpreting a remainder as a fraction, for example 16 ÷ 3 = $5\frac{1}{3}$

Opportunity for: explaining and discussing

Double-decker pizza

Time 5–20 minutes

Resources

- Number cards 1–15 or to suit your children (Resource sheets 1 and 2)
- ÷ and = cards (Resource sheet 8)
- Plastic fraction pieces or *Fractions* (Resource sheet 42)
- At least one other child

Check: does the child use key vocabulary?

remainder	half
left over	quarter
fraction	three-quarters
divided by	thirds
whole one	two-thirds

Teaching activity

'This game, **Double-decker pizza**, will help you to get better at making remainders into fractions when you work out division calculations. In this game you must remember to choose two numbers that will give you a remainder, otherwise you won't be able to have slices to make up your pizza!'

Choose number cards to suit your children, for example 2, 3 and 4, with cards 8–15.

Put the cards on the table face up in two groups: the 2, 3 and 4 together, and 8–15 on the other side of the table.

Put the fraction pieces on the table, face up. If you use the resource sheet you will need to make several copies of it and cut up the fraction pieces.

The game can be played cooperatively, with pairs trying to make two pizzas.

How to play

1. The players take turns to pick up two cards, one card from 2–4 to show how many to divide by (or the size of step on the number line) and the other from the 8–15 cards to show the number to put on the right-hand end of the number line.

2. The player reads out their number sentence: 'Fourteen divided by three.'
3. Everyone then works out the calculation, for example on their own empty number line.
4. The player then says their answer – in this case, four remainder two – and makes the remainder into a fraction – in this case two-thirds.
5. If everyone agrees that they are right, the player will win a two-thirds fraction piece.
6. The cards are put back on the table and the other player chooses two more cards – at least one of which must be different from the first player's. So if a player has just divided by three, the new player must divide by either two or four.

7. If children are competing, the winner is the first player to make a whole pizza (for a 5-minute game with two players). For a longer game, the winner needs to make two pizzas to make a double-decker.

? **Which fraction do you need to make to complete your pizza? So which number cards would be good to choose?**

? **Can you explain to me how you knew to choose those two numbers to complete your pizza?**

? **Could you have chosen different cards to get the same fraction answer?**

? **Is it always possible to make a whole pizza in two goes?**

? **How do you know?**

Variations

- Play with different number cards, for example a complete set of 1–20 or 30. For this game you need to have lots of paper circles and make your own fraction piece of pizza if it isn't on the resource sheet or in your plastic fraction set. (For example, if you divide by five you might need several fifths of circles.)
- Play with just a small selection of cards but have them face down (but still in the two sets). (This involves more luck but gives less opportunity to use reasoning to choose numbers to complete a pizza. This might give a child who is having more difficulty with the game a greater chance of winning. Allow at least 15 minutes for this game.)

? **Why can you never make a whole pizza in one go with these cards?**

? **Which number cards would you need to make a whole pizza in one go?** *(With two sets of cards so you could make, for example, 2 ÷ 2 = 1 so you wouldn't be making a fraction.)*

? **Using any numbers up to a hundred, write five calculations that would give the same size fraction part in the answer.**

Learning outcomes

By the end of this set of activities, children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- interpret a remainder as a fraction;
- interpret 'divide by' as a grouping calculation;
- work out a grouping calculation on a number line and show and name the fraction part.