Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: communicating mathematics in words and symbols

Resources

- Number cards and symbol cards (Resource sheets 1, 2 and 8)
- Number lines
- Cubes

Key vocabulary

other way around
array
multiplied by
divided by
divided between
count back
hops back

Teaching activity

Time 15–20 minutes

Explain to the child that the activity today will help them to learn more about ways in which we can rearrange numbers in a number sentence.

? **Which number sentences can you remember where you can rearrange the numbers and you still get the same answer?**

If the child needs support with this, show a + card and lay out an addition, for example: 3 + 4 = 7

? **What if we reverse the three and the four? Will we still get the same answer?**

Clarify that four plus three is also seven.

Let the child suggest a few more examples, such as five plus two and two plus five.

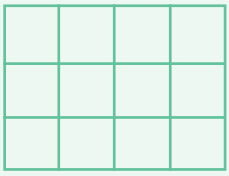
'So we can add numbers in any order and we still get the same answer.'

? **Can you make more than one multiplication number sentence with four, three and a × ('multiplied by') card?**

? **Does three multiplied by four give the same answer as four multiplied by three?**

? **Can you make something with these cubes or draw something to show me how you know you are right?**

Support the child to make an array and hops along a number line.



$3 \times 4 = 4 \times 3$

a hop of three, four times 3×4

a hop of four, three times 4×3

'If you make three hops of four you land on 12, and if you make four hops of three you land on 12.'

? **Can you think of some number sentences that don't have the same answer when you put them round the other way?**

If the child has difficulties with this, support them with some examples of subtraction number sentences.

'Twelve subtract four equals eight, but if you change around the twelve and the four to four subtract twelve, what answer do you get?'

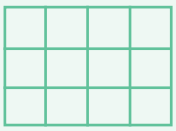
? **What division number sentences can you make with twelve and three?**

? **What do you know about the answers to the number sentences you have made?**

To support the child here, you could ask them to make a picture of what these division sentences mean.

$12 \div 3 = 4$

You can show this with an array, or as hops of three back from 12 along a number line, or twelve biscuits into packs of three.



? **What picture could you make to show three divided by twelve?**

Help the child to read the sentence 'three divided by twelve'.

'Suppose we thought of three biscuits divided into twelve pieces. Each one of those pieces is going to be quite small (a quarter of a biscuit) so the answer to three divided by twelve is different from the answer to twelve divided by three!'

? **Show me the operation cards that we can use to reverse number sentences and still get the same answer.** *(+ and ×)*

? **Which operation cards are the ones where reversing the number sentence gives a different answer?** *(– and ÷)*

Finish with cards 3, 6 and 18, asking the child to make a division calculation that would go with $3 \times 6 = 18$.

? **Using any two numbers, write some calculations that are the same the other way around, for example, 17 + 15 is 32 and so is 15 + 17. Which operations don't work with numbers the other way around?**

Spotlight 1

Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: exploring a real-life context

Tangerine arrays

Time 15–20 minutes

Resources

- Real or modelling material tangerines
- Counters or cubes
- Number cards (Resource sheets 1 and 2)

Key vocabulary

other way around
array
multiplied by
divided by
count back
divided into three
fraction

Teaching activity

'Today we are going to make lots of number sentences about these tangerines. We are going to work with division cards only, and we are going to make number sentences that will always mean that the tangerines are not cut up.'

'Let's start with twelve tangerines and a 12 number card.'

? **Can you choose two number cards that we can use to divide twelve into equal groups?** *(6 and 2, or 3 and 4 are more useful in this context than 12 and 1.)*

If the child is unsure about choosing numbers, you could lay out the tangerines in an array, or, instead of constantly handling real tangerines, you might want to make the array with counters.

Ask the child to describe the array.

'There are three groups of four tangerines or four groups of three tangerines.'

? **So can you choose two number cards now to go with this array?** *(3 and 4)*

? **Can you make two division number sentences with twelve, three and four?**

If the child cannot do that, or is confused about the order of the number cards, help them by making one number sentence and reading it out: 12 ÷ 3 = 4

'Twelve divided by three makes four groups.'

(Saying 'how many threes go into twelve' is potentially confusing, so keep away from 'goes into'. 'Three into twelve' is another confusing phrase.)

? **Do we need to cut up any of the tangerines to divide twelve by three?** *(No)*

'Show me how this division number sentence matches with the array.'

If the child can't relate the number sentence to the array, use the array to show how the twelve can be divided by three to give the answer four. Let the child split up the array and talk you through:

3 + 3 + 3 + 3 makes 12 and 3 × 4 makes 12.

If you start at 12 on a number line, you can hop back in threes.

12 – 3 – 3 – 3 – 3 = 0, so there are four threes in twelve.

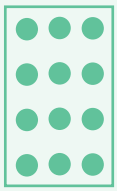
Twelve divided by three makes four.

? **Can we make a different number sentence with these same three numbers but still not have to cut up the tangerines?**

If the child is finding that difficult, go back to the array and ask them if there is another way of looking at the array of twelve.

? **What if we see the array the other way around?**

You can encourage the child to either move the array, or move around the table so that they see the array as four rows of three.



Help them to make the number sentence:

12 ÷ 4 = 3

'Twelve divided by four makes three.'

'This relates to the array that is four rows of three.'

If the child wants to split the twelve into different numbers, for example seven and five, remind them that with multiplication and division we make equal groups. Seven and five are not equal groups.

If the child makes a sentence such as 3 ÷ 12 = 4, or 3 ÷ 4 = 12, ask them to think of the picture which those sentences would make.

? **Do these number sentences go with our array of tangerines?** *(They don't.)*

? **Can you read those sentences to me?**
'3 ÷ 12 = 4 is three divided by twelve, so if it was tangerines and we had three tangerines to divide between twelve people, we would have to cut the tangerines – and the answer wouldn't be four! The answer would be a fraction, just a little bit of a tangerine.'

? **Can you change the cards around so that we can use all three numbers and it makes a division sentence where we don't have to cut up the tangerines?**

If the child doesn't write sentences such as 3 ÷ 12 = 4, you need to mention it to them anyway to make sure that they have considered calculations like this.

? **Can you use these same three cards to make a division number sentence that would mean we would have to cut up the tangerines?**

? **Which number would we start with that would mean that we didn't start with many tangerines?**

'3 ÷ 12 = 4 is three tangerines to be divided by twelve so we would need to cut the tangerines. The answer isn't four, so we must be very careful when we are writing division calculations that "go" with an array.'

If you think the child might be able to cope with it, you could ask them to draw a picture of what three tangerines divided by twelve might look like.

'If you cut them into twelve bits for twelve people, each person would just get a quarter of a tangerine. So the answer isn't four, it is less than one whole tangerine, just a fraction, so 3 ÷ 12 = 4 is wrong.'

? **What did you learn today about rearranging the numbers in division calculations?**

Spotlight 2

Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: developing mental images

More than one, less than one

Time 15–20 minutes

Resources

- Sweets or counters
- Bead string
- Number lines

Key vocabulary

other way around
groups of
divided by
count back
count forward
hops of
fraction
more than one
less than one

Teaching activity

'Today we are going to do some division calculations and we are going to see which ones give us answers of more than one and which ones give answers of less than one.'

? **What do I mean by a number 'more than one'? What about a number less than one?** *(Talk about a fraction of a whole apple – a bit that is less than one whole one, maybe a half or a quarter.)*

? **What picture do you have in your head if I ask you to think about twenty divided by four?**

'You can use anything on the table to work that out, or you can draw the picture that is in your head.'

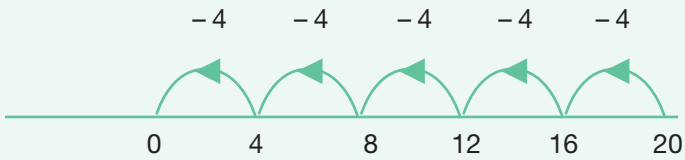
If the child seems stuck, ask them to read the sentence 20 ÷ 4, 'twenty divided by four'.

? **Shall we start with twenty of these sweets or do you want to use a number line or a bead string?**

If the child starts sharing 'one for me, one for you', accept this for now, but make a note to move them on from that later.

If the child shares the sweets into four groups of five, ask the following question.

? **Is there another way to divide twenty by four? What if we put the sweets in groups of four? How would we show that on a number line?**



Help the child to see that we can start at 20 and count back in steps of four. Five steps of four get us to zero. Or we could count up from zero in steps of four until we get to 20.

? **So what is the answer to twenty divided by four?**

If the child needs more support, make an array of five by four and show how it can be divided into five groups of four.

'So twenty divided by four is five. That calculation gives us an answer of more than one.'

Record the calculation with cards or on paper.

20 ÷ 4 = 5 This answer is more than one.

? **What if we reverse the twenty and the four?**

4 ÷ 20 =

? **Can you read that number sentence?** *(Four divided by twenty.)*

? **What picture do you have in your head?**

Understandably, some children might be quite confused at this!

? **How many sweets does that tell us to start with?**

Explain that it means four sweets divided into twenty!

? **If we divided four sweets into groups of twenty is each person going to get a whole sweet?**

'If we write the division this way around, the answer will be less than one.'

? **What did you learn today about what happens when you change the numbers round in divisions?**

'Write a division with the numbers fourteen, two and seven so that the answer will be more than one.'

14 ÷ 2 = 7 or
14 ÷ 7 = 2

If the child makes an error, note that for next time.

Spotlight 3

Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: making decisions

Tricky treats

Time 15–20 minutes

Resources

- Number cards (Resource sheets 1 and 2)
- ×, ÷ and = cards (Resource sheet 8)
- Cubes

Key vocabulary

other way round
array
multiplied by
divided by
count back
hop back

Teaching activity

'I'm going to play a game with you. I'm going to show you some number sentences and I might pretend to read a number sentence wrongly to trick you. You have to tell me if I am reading a number sentence wrongly or if I have put the wrong answer.'

Show two each of the cards 4, 5 and 20, and ÷ and × operation cards. Make these two number sentences: 4 × 5 = 20 and 5 × 4 = 20.

? **Am I tricking you?**

? **Are these number sentences both correct?**

Both of those number sentences are right and give the same answer.

Make two division number sentences using the cards to display them:

20 ÷ 4 = 5 4 ÷ 20 = 5

? **Are these number sentences both correct? Explain how you know.**

? **Can you make a picture of these sentences in your head and sketch it?**

If the child cannot see that 4 ÷ 20 = 5 is incorrect, support them in making pictures either on paper or with cubes, and ask them to read the sentences carefully.

'20 ÷ 4 = 5 can be read as twenty divided by four. That makes five hops of four along a number line, or an array that is four rows by five columns.'

'4 ÷ 20 = 5 can be read as four divided by twenty.' (You could try reading it as twenty divided by four to see what reaction you get.)

'Four cakes divided up into twenty pieces. The answer to that isn't five. The pieces would be small and the answer would be less than one. I was tricking you. Four divided by twenty equals five is wrong.'

? **Can we put the three numbers, four, five and twenty, in a different order to make a correct number sentence?**

Support the child to re-order the cards to: 20 ÷ 5 = 4 or 20 ÷ 4 = 5.

'Let's make some tricks for the rest of the class. Let's make some divisions that are right, and some that are wrong, and we will see if they can work out which ones are wrong.'

(50, 25, 2) (30, 10, 3) (35, 7, 5)

Are these right?

50 ÷ 2 = 25 30 ÷ 10 = 3 7 ÷ 35 = 5

2 ÷ 50 = 25 10 ÷ 3 = 30 5 ÷ 35 = 7

Spotlight 4

Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: linking mental images and language

Draw the picture

Time 10–15 minutes

Resources

- Several sets of number and symbol cards (Resource sheets 1, 2 and 8)
- Cubes

Key vocabulary

other way round
array
multiplied by
divided by
count back
hop back

Teaching activity

'Today we are going to do some drawings of pictures from our heads that go with number sentences, to help you to understand more about division and multiplication.'

? **If we have the cards 3, 5 and 15, which multiplication sentence could you make?**

Support the child to make 3 × 5 = 15.

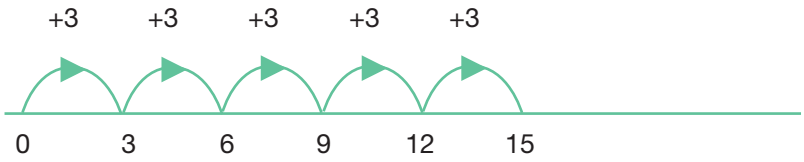
? **Can you put the numbers in a different order but keep the sentence correct?**

Help them to make 5 × 3 = 15.

? **What is the picture that is in your head to go with 5 × 3 = 15?**

While the child draws, you could draw an image as well, perhaps one that the child rarely draws or uses.

'Five hops of three, so this image is three multiplied by five, or three five times.'



'Five rows of three, or look at it the other way round to make three rows of five.'

Accept a range of drawings. (A really weird one might just give you insight into what the child is thinking!) Stress the point that you only want pictures that go with five, three and fifteen.

> If the child makes something like 15 × 3, show how that doesn't fit with the pictures.

Change the cards around again to make the two different multiplication sentences, making sure that the child can match the sentences to the various pictures.

Can you use two division signs to make two division sentences that are right and that fit with our pictures?

What is it that we can do with multiplication calculations that doesn't work with division?

Help the child to read the division sentences carefully, so that 15 ÷ 3 can be read as 'how many threes make fifteen', or 'fifteen divided by three'. To fit with the number line hops, try to move children from saying 'shared between' to saying 'divided by'.

If a child makes something like 3 ÷ 15 = 5, help them to read it.

Does it fit with any of our pictures? *(It shouldn't!)*

Clarify that this sentence means three divided by fifteen, or three divided into fifteen.

'This does not fit with the pictures we have been using. Each part would be just a fraction – less than one.'

Explain to me what goes wrong if we try to swap numbers around in a division calculation.

Can you give me an example to show me what you mean?

Can you draw a picture to help me to understand?

What helped you to learn something new today?

Spotlight 5: a learning check

Assumes that the commutative law holds for division also, for example assuming that 15 ÷ 3 = 5, so 3 ÷ 15 = 5

Opportunity for: discussing and explaining

Fractions in the bin

Time 15–20 minutes

Resources

- Two or more players
- Counters or rewards
- *Fractions in the bin game board* (Resource sheet 40)
- *Fractions in the bin game cards* (Resource sheet 41)

Check: does the child use key vocabulary?

other way round	divided by
array	count back
multiplied by	divided into groups of three

Teaching activity

Note: Resource sheet 41 needs to be turned to landscape format to match these instructions.

The top three rows of cards go with the pictures and there is one blank card that you can use to challenge a child to write another card.

The cards in the fourth row are reverses of the arrays, for example 9 × 2 isn't a picture of the top picture on the board unless you view it from the side so that it becomes nine rows of two, nine two times, or nine multiplied by two. You might want to keep these cards for when children are more confident.

None of the cards in the fifth row matches any of the pictures, so again, you might want to keep these for later.

'We are going to play a game, **Fractions in the bin**, with cards on this board. Some of the cards go with the pictures on the board and some don't. So 2 × 9 would go with this picture, but 9 ÷ 18 won't go with that picture so it goes in the bin.'

Put out just the cards you want to play with (probably just the first three rows at first) face up on the table, and a board for each pair of players. Give players a moment to look at the pictures and to count rows, and so on.

How to play

1. Each player starts with two rewards.
2. They take turns to take one card, read it carefully and place it near the right picture. If everyone agrees that they are right, they win a reward.
3. If they are wrong, they can think again about it and try to put it in the right place. If they are still wrong, they have to give one of their rewards back!
4. If a card doesn't match any of the pictures, this card is put in the bin. The player will win a reward if they are right that it doesn't match a picture.
5. The winner is the player with the most rewards at the end.

Variations

- Each child has their own game board and a set of cards (maybe five each). A timer is set and they try to put their cards in the right places as quickly as they can. When they have placed them all, a note is taken of the time and, the next time they do the activity, they race themselves to see if they can do it faster. For every wrongly placed card, ten seconds is added to their time!
- The players make their own game board of pictures and their own set of about ten cards, right ones and wrong ones. They could include written questions such as 'How many twos make eighteen?' They then swap sets of cards and race to put them in the right place. (This needs close supervision to make sure that the cards and pictures make sense!)

? **Explain to me how you knew that picture matched that card.**

? **Why does that card go in the bin?**

? **Can you read that card and tell me what picture it makes in your head?**

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand that division calculations give a different answer if the numbers are reversed;
- write a division calculation that links with a multiplication calculation;
- understand that multiplication number sentences can be reversed and still give the same answer.