Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions

Resources

- Tape measure
- Number lines
- Calculator

Key vocabulary

estimate	almost
guess	nearly
rough answer	

Teaching activity

Time 15–20 minutes

Explain to the child that today's activity will help them to make an estimate of a calculation.

'Tell me what you think making an estimate means.'

? Give me an example of making an estimate.

If the child is unsure about this, give an example.

'When we are measuring, we often make an estimate or a guess. So I could estimate how far it is across our table. My guess is that it is about (eighty centimetres/or other). My guess probably isn't the exact measurement – it is only an estimate, or a guess, or sometimes we can call it a rough answer.'

? How could we find out if my guess is right?

Help the child to measure across the table.

'My estimate was close/not very close. But it was just a guess: an estimate.'

'Estimating can help us when we are doing calculations too. So if we know that a hundred divided by two is fifty, if we have to work out a hundred and seven divided by two, we can make an estimate of the answer. The answer must be somewhere close to fifty.'

Let the child work it out on a calculator.

'So fifty-three point five is quite close to fifty. Fifty was our estimate. Fifty-three and a half was the exact answer.'

You might find it helpful to use a number line to 100, marking 100, 50 and 25.

? Which do you think would be the best estimate of fifty-six divided by two: twenty-six or six?

Help the child to think through dividing fifty by two to get to twenty-five, so twenty-six is a much closer estimate than six.

You might want to record some calculations to take back to class or to refer to later on in this work. For example:

An estimate of 17 × 2 is about 25.

Or 17 x 2 is about 30.

Which is closer to the actual answer, 25 or 30? How do you know?

Help the child to explore their thinking. Again, a number line might help. Show how 17 × 2 = 34, so 30 is a much closer estimate than 25 on the number line.

If the child seems lost and is not grasping what an estimate is, you might find that it helps to go completely over the top and write some calculations that they will see are nonsense!

12 × 3 is about 14 562

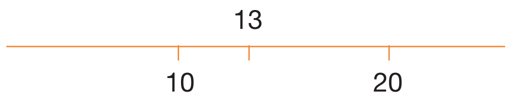
10 ÷ 2 is about 376

10 × 2 is about 968

How do you know these are not close estimates?

'So when we make an estimate, we don't need to get the answer right, but we do need to make a close guess to try to find an estimate that is close to the actual answer.'

If we wanted to make an estimate of thirteen multiplied by nine, which would be the closer estimate: 10 × 10 = 100 or 20 × 20 = 400?



Again, a number line would be useful. Thirteen can be seen to be quite near ten, but further from twenty. Nine is also near ten, so ten multiplied by ten is likely to give us an estimate which is nearer the actual answer than twenty multiplied by twenty.

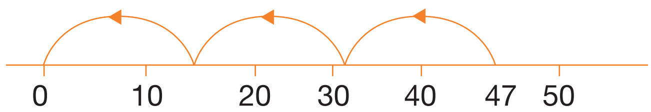
Support the child to calculate thirteen multiplied by nine in any way they want. For example, 10 × 9 = 90 added to 3 × 9 = 27 that is 117 altogether.

So was our estimate close to the answer?

If we wanted to make an estimate of forty-seven divided by three, do you think the answer would be nearer to ten or nearer to a hundred? Why?

You might want to find 47 on a number line and help the child to think about roughly what size each of the three hops back might be.

If we make a guess at the size of each of the three hops back from 47, about how big is each hop? Make a guess.



Is each hop nearer to a hop of ten or a hop of a hundred?

Is each hop a little bit bigger than ten or a little bit smaller than ten? *(Bigger)*

47 ÷ 3 is somewhere around ten, but a bit more than ten.

We worked it out and it is 15 and a bit, so we made quite a good estimate.

? What shall we record from today's lesson that is really important to remember for next time?

(*Note:* Children who think that mathematics is only about getting the one right answer can be confused by estimating – presumably because we are accepting their guesses and saying those are 'right'! And we tell them not to work it out exactly at first! So you might find you have to repeat this lesson and, crucially, emphasise making an estimate of a calculation *every time* we do a calculation, and then using the estimates after calculating to check that our answer is a reasonable one.)

Spotlight 1

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: solving real-life problems and reasoning about numbers

Estimating with multiplication tables

Time 15 minutes

Resources

- Cubes
- Ten paper 'tables '
- Number lines
- *Multiplication grid 1* (Resource sheet 35)
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

'We are going to do some estimating today. Sometimes we will be working out answers exactly, but first we are going to make a good guess, an estimate, and we are going to do just a little rounding of numbers.'

(*Note:* Take care when talking about rounding. During trialling, when asked to round numbers, some children who were asked to find an approximate calculation for 49 × 2 stuck so closely to rules of rounding that they rounded the calculation to 50 × 0! Not a good estimate!)

'The teacher has thirty-two children in her class and they sit eight to a table. Shut your eyes and think about those numbers and try to see the children in the class and the tables.'

Show the pieces of paper representing tables. 'Eight sit at this table, eight more at this table...'

? About how many tables do you think they will need?

Give the child time to think. Let them choose something to help – but not a calculator. They could put eight cubes around one of the pieces of paper and look at how many cubes are left for the other tables.

If the child seems stuck, you could try asking them to think of an easy number near thirty-two, probably thirty.

'We can round thirty-two down to thirty.' Use a number line to show how close 32 is to 30.

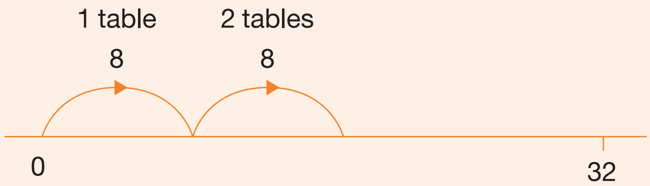
? **About how many eights are there in thirty? So do you think an estimate of five or ten tables is the closest?**

Or talk through the fact that ten tables with eight to a table would seat eighty children!

If the child knows that 4 × 8 = 32 and can see that four tables are needed, move on.

If the child needs more help, let them do a recording in any way they choose to show how they worked out the calculation.

If they are stuck, suggest they use a number line and see how many hops of eight they need, starting either from zero and counting up, or starting from 32 and counting back.



'You need four hops of eight to get to 32. So five was the closest of the two estimates.'

'We found 4 × 8 = 32 on the multiplication grid.'

You can record for the child. Support the child to work mentally at first, using a number line and/or a multiplication grid, and later let the child use a calculator to try each calculation.

'We know that thirty-two divided by eight is four: 32 ÷ 8 = 4.'

? **What if the tables only seated six children? Would thirty-two divided by six give a bigger or a smaller answer than thirty-two divided by eight?**

(Record what the child thinks. The answer is larger because steps of six are smaller than steps of eight so you need more of them. The child might explain that you would need more tables if they only seated six children.)

? **What if the teacher only had twenty-four children and it was eight to a table?**

(Record for the child. The answer is 'smaller' – the teacher would need fewer tables with fewer children.)

If the child is struggling with this, you will need to repeat this lesson, because being able to reason about numbers in order to make estimates is so important. You can use numbers to suit the child.

Also try some more over-the-top calculations.

Do you think 100 ÷ 2 = 3? How do you know?

Is this right: 94 ÷ 6 = 5? How do you know?

? **Is eighteen multiplied by nine larger or smaller than eighteen multiplied by eight?**

In a real-life context that is:

Are there more marbles in nine bags or in eight bags?

What shall we record today that is really important to remember?

Spotlight 2

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions

Rounding to estimate

Time 15 minutes

Resources

- Number lines
- 100-square
- Bead string
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

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'We are going to do some more estimating today, using rounding.'

Can you remember what rounding is?

How could you round forty-nine multiplied by two to help find an estimate of the answer?

Clarify that forty-nine can be rounded to fifty – but we wouldn't round the two!

What is fifty multiplied by two?

Do you think that sixty is a good rough answer for forty-nine multiplied by two? Why not?

> You could show two hops of forty-nine on a number line. Or show 49 and 50 on a hundred square, or on a bead string.

If I have fifty-nine pence, have I got nearly sixty pence or nearly seventy pence? How do you know?

How could we estimate fifty-nine divided by three? 59 ÷ 3 = *(60 ÷ 3 = 20)*

Can you make up a number story for fifty-nine pence divided by three? *(Fifty-nine pence divided into three equal groups so that three children can have one-third of the money each.)*

You could use a bead string to show the rounding to sixty, then find how big the three hops back from 60 to 0 are.

But there is only fifty-nine pence! So there isn't enough for each child to have twenty pence. Twenty pence was only an estimate – not the actual answer. The estimate of twenty pence helps us to check the actual answer.

? **What do you think the actual answer is? It must be very close to twenty pence.**

Estimating helps us to check our answers.

? **When we work out forty-seven multiplied by two, which number would we round up or down to help us find an estimate?** *(Forty-seven probably best rounded up to fifty.)*

? **Would you need to round the two? Why?**

? **How could we find an estimate for five hundred and ninety-nine multiplied by two?**

? **Would you round the two?**

? **What about seventy-eight multiplied by thirteen? What might be an estimate for that? How did you decide?**

On a number line you could show how 78 is close to 80 and 13 is close to 10.

'This time we could round both numbers to make the estimating easier to do. We rounded 78 up to 80 and we rounded 13 down to 10.'

Let the child work out 80 × 10, record the answer, then work out on a calculator the actual answer to 78 × 13. Mark both numbers on a number line.

Give another example of rounding one number up and one down, for example: 73 × 9.

Round seventy-three down to seventy. Round nine up to ten.

$70 \times 10 = 700$

? **What shall we record that is important to remember for next time?**

Support the child as they reflect on what they have learned.

Keep this recording for the next session.

Spotlight 3

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: making decisions and explaining

In-between

Time 10–20 minutes

Resources

- Selection of number lines
- 100-square
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	which is closer?

Teaching activity

'Let's look back at what we wrote last time. We wrote down something very important to help us remember ways of finding estimates.'

Go over what you wrote.

'Today we are going to look at another strategy that can really help us when we want to estimate an answer. We can do two estimates so that we know that our actual answer must be somewhere between those two estimates.'

? **How could you estimate the answer to two hundred and fifty-seven multiplied by three?**

Support the child to see that you could round 257 up to 260 or down to 250. (We won't round the three!)

> If the child is struggling with this rounding, use a number line from about 250 to 300 to show the rounding to numbers with zeros, and extend the line to about 800 to show where the rounded answers and the actual answer fit on the line.

257 x 3 → round up to 260 x 3 = 780

(The actual answer is between these.)

257 x 3 → round down to 250 x 3 = 750

Making one estimate above and one below the actual answer (771) can help us to be sure our answer is correct.

So if you work out 257 x 3 and get 1078 you know you are wrong!

Ask the child for another three-digit number to round both up and down so that the answer is between the two, for example 386 x 2. *(This could be rounded up to 390 x 2 = 780 and down to 380 x 2 = 760. The actual answer is 772.)*

? **What did you learn today?**

(You can make estimates every time you calculate anything because they help you to get more answers correct. **Estimate, calculate, check.**)

? **How could you work out an estimate for 555 × 5?**

Spotlight 4

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: generalising about numbers

More or less?

Time 15 minutes

Resources

- Squared paper
- Cubes
- Selection of number lines
- 100-square
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

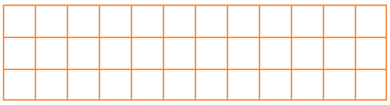
Teaching activity

'We are going to do some estimating answers today, and we will be trying to work out whether the estimates we make will be more or less than the actual answers. We are going to start with multiplying.'

? **Which is more, three multiplied by twelve or four multiplied by twelve? How do you know?**

? **Can you choose something on the table to show me how you know?**

Encourage the child to make or draw an array on squared paper or with cubes. You could record this.



3 multiplied by 12

Three multiplied by twelve means twelve columns of three.

Four multiplied by twelve would need twelve columns of four, so 4×12 is more than 3×12.

If the child has forgotten about arrays, make some on squared paper with them and stick them up somewhere so that the child can be reminded about them over the next few days and weeks.

Try some more examples.

? **Which is larger, twenty-seven multiplied by five or twenty-seven multiplied by four? Can you tell me how you know that?**

Let the child work out the actual answers using a calculator, because the point of this lesson is to be able to have lots of experience of comparing answers. Using calculators can take the worry out of finding answers and it allows children to concentrate on understanding the mathematical relationships.

Record for the child:

27×5 is more than 27×4.

16×6 is more than 16×5.

46×9 is more than 46×8.

You could use colour to emphasise the five and the four in the first calculation, then another colour to emphasise the six and the five, and so on.

? Can you see a pattern in the coloured numbers?

Look at each example carefully and try to move to making a generalisation.

27 × 5 is more than 27 × 4.

16 × 6 is more than 16 × 5.

46 × 9 is more than 46 × 8.

When we multiplied these, we found a pattern: the larger the numbers you multiply by, the larger the answer.

27 x **5** is more than 27 x **4**.

To support the child, you could use numbers that are further from each other, for example 8 × 5 is less than 8 × 10.

Let the child draw or make arrays and calculate the answers, looking for the pattern that the larger the numbers are, the larger the answer will be.

? Can you tell me how you know that the larger the numbers you multiply by, the larger the answer will be?

'Let's try to see if we can find a pattern when we divide.'

? Predict what might happen when we divide eight by *two* and when we divide eight by *four*.

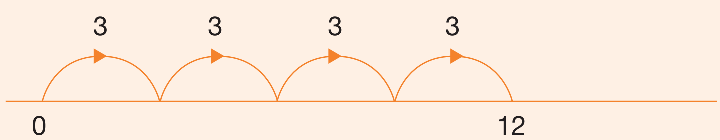
? Which will give the smaller answer: eight divided by two or eight divided by four?

? Thirty-two divided by two or thirty-two divided by three?

? Twelve divided by three or twelve divided by four?

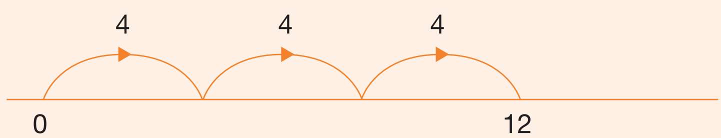
? Twenty-four divided by six or twenty-four divided by eight?

Remind the child that twelve divided by three means how many threes make twelve, and we can show this on a number line as 12 with equal steps of three. Each step will be a step of three and there are four of them.



Twelve divided by four means how many steps of four are there in twelve?

There are three steps of four in twelve.



12 ÷ 3 = 4
12 ÷ 4 = 3

Support the child to moving towards making a generalisation.

Which will give the smaller answer:

$32 \div 2$ or $32 \div 3$?
$12 \div 3$ or $12 \div 4$?
$24 \div 6$ or $24 \div 8$?

The smaller the number you divide by (the step size), the larger your answer.

The smaller the size of the steps along the number line, the more of them you need (a larger number of steps).

The larger the number you divide by (the step size), the smaller your answer.

'Let's check our prediction.'

Let the child explore, using a calculator to check the generalisations about multiplication and division. You record for them.

You could take the generalisations back to the class.

(*Note:* This kind of activity using a calculator can help children to develop a strong understanding of place value.)

This would be a good time to play the game **In one move**, which can be found on page 16.

Spotlight 5

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: discussing mental methods

Be careful!

Time 15–20 minutes

Resources

- Cubes
- Squared paper
- Selection of number lines
- Calculator

Key vocabulary

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

'This lesson is about making estimates of calculations first, then using the estimate to find an exact answer.'

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? **Can you think of a way of estimating nineteen multiplied by four?**

? **Make your estimate and tell me how you've done it.**

A calculation the child might suggest is 20 × 4.

Encourage children not to round the four to five, because that would make the estimate a long way from the actual answer. They can multiply by four by doubling the twenty to forty and double again to get to eighty, or use another mental method they like.

? **Can you use your rough answer, your estimate, to find the exact answer in your head?**

Listen carefully to what the child says. They may be making the error shown below.

Note: In trialling, children sometimes used mental methods more appropriate for addition and subtraction calculations. So this Spotlight focuses on this.

If the child says anything along the lines of: 'We rounded up from nineteen to twenty so to find the real answer I can just take away one, so the actual answer is 80 – 1 = 79' then the child is using addition and subtraction mental strategies! You might want to record these because they do need talking through.

19 × 4 is close to 20 × 4 = 80

So to find the exact answer I can do 80 – 1 = 79

because we added one to get from 19 to 20.

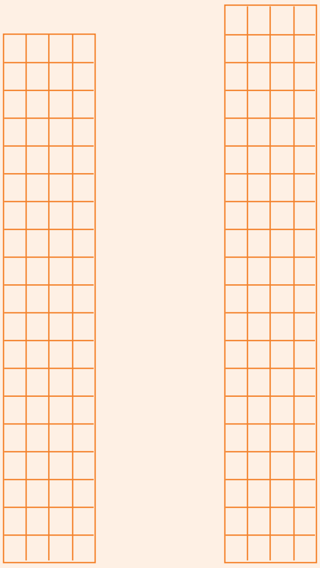
But that is wrong.

19 × 4 = 76 not 79.

Where did we go wrong?

Let the child work out the actual answer using a calculator and record the results for them.

Support the child to use squared paper or cubes to see the difference between an array of four columns of nineteen and four columns of twenty. There needs to be one extra row of four, four more cubes on the array.



'There is one extra row of four cubes here.'

19 × 4 20 × 4

'So subtracting one was wrong. I think you probably meant to subtract one row of four. You were using a mental method used with addition and subtraction, not with multiplying and dividing.'

'When we estimate in multiplying and dividing, each time we round up or down, we add on or take off columns or rows.'

Give another example, such as an estimate for 26 × 3.

? **When we are estimating calculations with multiplication and division and then trying to find the actual answer, is it okay to use the same mental methods that we use for addition and subtraction?** *(No)* **How do you know that?**

? **What shall we write down for today that is important to remember?**

Why do you think it is important to be able to make estimates?

Encourage children to make estimates of every calculation they do!

You could put the following on the wall:

Estimate
Calculate
Check
Every time!

Spotlight 6: a learning check

Is not confident in making reasonable estimates for multiplication or division calculations

Opportunity for: explaining and discussing

Which side?

Time 10–25 minutes

Resources

- At least one other child, four is ideal
- Counters
- Bag with number cards 1–9 (Resource sheet 1)
- Another bag with two cards: 'more than' and 'less than'
- *Spinners* (Resource sheet 44)
- *Blank spinners* (Resource sheet 13)
- Paper clip and pencil
- Selection of number lines
- Calculator

Check: does the child use key vocabulary?

estimate	about right
guess	close but too big
rough answer	close but too small
almost	round up/down
nearly	

Teaching activity

'This game, **Which side?**, will help you to get better at estimating, and we are going to use a number line to put our estimates on and compare them with the actual answer.'

You might want to put on display the recordings from previous Spotlights of things that needed to be remembered.

Players can work cooperatively in teams or in pairs, or they can compete on their own against another player.

The spinner is worked by trapping the paper clip in the middle of the spinner with the point of a pencil. With the other hand, flick the paper clip around. If the paper clip lands on a line, that player chooses which number they want.

Spinners work best if they are copied onto card.

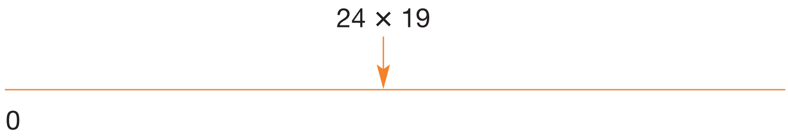
Before you start, decide which spinner you are going to use. If you cut out your spinner, be sure to cut in along the boxes in order to leave enough space for the spinner to be held still while it is being used.

Blank spinners (Resource sheet 13) can be used for any numbers you choose.

How to play

1. A player takes two digit cards out of the bag and makes a two-digit number, for example 24. This number is then recorded and is used for both teams. The digit cards are put back in the bag.
2. Another player, on any team, flicks the paper clip to choose the operation and number, for example × 19.

3. Each team then records the calculation on paper, in this case 24×19, and someone draws a number line and marks the calculation.

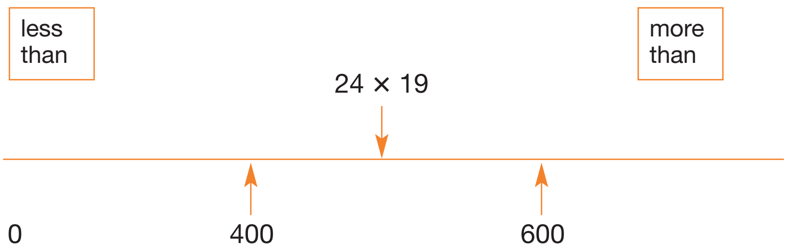


4. Each team then takes one card out of the other bag and shows it to the group.
5. The team with the 'more than' card must make an estimate of the calculation 24×19 that will give an answer which is more than the actual answer. The team with the 'less than' card must make an estimate which is less than the actual answer.
6. Each team then works quietly together to work out their estimate.
7. When everyone is ready, the teams take turns to tell everyone about their estimate.

   'We had to make an estimate that would be more than 24×19, so we chose 30×20 which is 600.'

   'We had to make a rough answer less than the actual answer, so we estimated 20×20 which is 400.'

   The teams can challenge any estimate which they think isn't sensible, for example 40×30 would not be a sensible estimate for this calculation.
8. Both teams can put their estimates where they think they go on the number line.



9. Then everyone has to work out the actual answer. (Calculators will speed this up and help to take the stress out of the game.)
10. Each team should keep a record of their score. For each round they get:
    ten points for a sensible estimate;
    ten points for calculating their estimate correctly;
    ten points for estimating 'more than' or 'less than' correctly;
    a ten point bonus for calculating the actual answer without a calculator.
11. Then a new two-digit number is made and another number selected on the spinner.

? **How did you work out that your estimate would be more than the actual answer?**

? **Whose estimate was the closest to the actual answer?**
How do you know?

? **What if that team had used (a number) instead of (a number)? Would their answer have been closer to the actual answer or further away?**

? **If we draw that number line again, could we position our estimates a bit more accurately?**
(Some estimates will be much closer to the actual answers, depending on the numbers chosen.)

? **Which team got the closest to the actual answer?**

? **What shall I record for you to remember for next time?**

Variations

- Play as **Who is the closest?** Players compete to find an estimate as close as they can to the actual answer. (So you don't need to play with the 'more than' and 'less than' cards.)

  The winning team scores ten points, or if they are both the same distance on the number line from the actual answer, they score ten points each. (You will need to help the players to decide who is the closest when they have two different answers. Encourage the players to round numbers to fives as well as numbers with a zero at the end, so fifty-four might be 'rounded' to fifty-five to give a closer estimate.)

  (*Note:* Watch out for children rounding all the numbers! On Spinner 2 hopefully no one will round × 4 to × 0!)

- Play **Who is the closest?**, as above, but also play with the 'more than' and 'less than' cards. This can be very tricky, and children will realise that if they have the 'more than' card, but the numbers are, for example 21 × 51, they will be a long way from the actual answer if they round up to 30 × 60.

? **Can you think of a way to estimate 21 × 51 that might be closer?** *(For example 25 × 50)*

? **What if you round one number up and the other number down? Does that give you a closer estimate?** *(It can do.)*

? **Tell me about an example when this happens.**

- Make three-digit numbers at the start.
- Use the blank spinner to make up your own numbers to multiply and divide. (The game can be played by estimating with addition and subtraction but clarify with the players that they must be careful in the way they go from their estimate to the actual number. What we do for addition and subtraction needs rethinking to find a method that might work for multiplication and division!)

Learning outcomes

By the end of this set of activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- make a reasonable estimate of a multiplication or division calculation including calculations with fractions;
- compare two multiplications or divisions and know which one will give the larger answer;
- understand that an estimate is a rough answer;
- make generalisations about the effects on numbers of multiplying and dividing by both numbers more than one and numbers less than one;
- understand that it helps accuracy to make an estimate for all calculations that cannot be calculated mentally immediately;
- understand that in real problem-solving we have to make sensible decisions;
- position estimates and actual answers on a number line.

In one move

Time 15–20 minutes

Is not confident in making reasonable estimates for multiplication or division calculations

Resources

- Children in pairs (or use with the whole class)
- Calculator for each child

Check: does the child use key vocabulary?

number pattern
more than
less than

How to play

1. Set a task, for example:

   'Key in 200. In one move (one operation, either $\boxed{\times}$ or $\boxed{\div}$, $\boxed{\text{a number}}$ and the $\boxed{=}$ key) make your display read 20.'

   Children can press cancel at any time if they want to – but not so that they can just key in 20!

2. Ask children what they did. You can record for them.

   > If you multiply 200 by 2 the answer is 400 so the answer is bigger than the 200 we started with.
   >
   > But, if you multiply the 200 by a number less than 1, for example 0.5, you end up with an answer (100) that is less than the 200 you started with.
   >
   > So using the x key doesn't always make the answer larger.

3. Repeat with the $\boxed{\div}$ key.

4. Once children are used to the game, you can introduce a scoring method, maybe 0.1 for each time a player does the task 'in one move'.

5. Encourage children to look for patterns.

$10 \times 0.1 =$	$100 \div 0.2 =$
$100 \times 0.1 =$	$10 \div 0.2 =$
$1000 \times 0.1 =$	$1 \div 0.2 =$

Variations

- Encourage children to make up challenges for each other, for example, start with 10 on the screen and just use the division key. Can you in one move make the screen read 100?
- Challenge children to make a long pattern using similar numbers, for example:
  If you start with 3000×0.3, can you make patterns using 300, 30 and 3? What if you start with 300 again but this time $\times$ by 0.1?

Learning outcomes

By the end of a set of related activities children should be able to:

- tackle related learning tasks with increased motivation and confidence;
- use and understand connected mathematical vocabulary;
- understand the effects of multiplying and dividing by numbers less than one.