Appendix 1

Tracking children's learning chart – addition and subtraction

The tracking charts act as an initial diagnostic tool and a background framework for the sets of teaching materials referenced to common misconceptions and errors.

Teaching unit codes are referenced against the errors and misconceptions listed in the chart, for example *1 Y6 +/–*

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Year 6 key objective Carry out column addition and subtraction of numbers involving decimals (NNS *Framework for teaching mathematics*, Supplement of Examples, Section 6, pages 49, 51)

Associated knowledge and skills	**Errors and misconceptions**	**Questions to identify errors and misconceptions**	**Teaching to address the errors and misconceptions**	**Next steps in moving towards the key objective**
Apply knowledge of the number system to enable efficient counting of a large number of objects. Add and subtract multiples of ten, a hundred and a thousand. **1 Y6**	Has inefficient counting strategies and/or insecure understanding of the number system. ***1 Y6 +/–***	Imagine you have a money box containing 2p and 1p coins. What do you think would be a good way to count these quickly to find out how much money there is? What is 60 + 20? … 60 + 30? … 60 + 40? What changed when you found 60 + 40? What is 40 + 40? … 400 + 400? Which answer is the larger? How is the calculation 40 + 400 + 4000 different from the others? What is 60 – 20? … 600 – 200? … 6000 – 2000? Explain how you worked these out. What is 6000 – 200? … 6000 – 20?	Practical opportunities to develop efficient counting strategies for a range of objects, for example coins, cubes, conkers, collectable cards, stickers. Count forwards and backwards in tens, hundreds and thousands from different starting points, including starting numbers that are not multiples of ten or a hundred. Use an empty number line to support this development. Order multiples of a hundred and a thousand.	Carry out simple calculations that involve crossing the boundary from hundreds to one thousand and vice versa, supported by an empty number line and extending this to a visualised image to develop mental calculation.
Give an estimate by rounding, to determine whether the answer to a calculation is sensible. **2 Y6**	Rounding inaccurately, particularly when decimals are involved, and having little sense of the size of the numbers involved. ***2 Y6 +/–***	Is 26 nearer to 20 or 30? Is 271 nearer 270 or 280? Is 1.8 nearer to 1 or 2? Draw a sketch to illustrate your answer and explain how you know.	Use number squares and/or number lines to consider the order and comparative value of numbers to support rounding.	Consider pairs of items from a catalogue and ask child to estimate whether a £10 (or £20, etc.) note would be enough to buy both the items?

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Associated knowledge skills	Errors and and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Partition whole numbers and decimal numbers, and add and subtract the constituent parts and recombine to complete the calculations. **3 Y6**	Has difficulty in partitioning numbers with zero place holders and/or numbers less than one, for example partitioning 0.45 as 0.4 and 0.05. ***3 Y6 +/–***	What is the value of the digit 1 in 3010? … 201? … 6.1? What do you notice about the value of the digit 2 in 4.2? … 0.2? … 0.25? Write the following statements: 507 is equal to 50 + 7 7403 is equal to 7000 + 40 + 3 0.75 is equal to 0.7 + 0.05 Which are true? How do you know?	Use place value (arrow) cards to partition and recombine numbers, including numbers with zero place holders and decimals, for example 10.85 and 18.05.	Provide or ask the child to select a calculation using numbers previously partitioned and ask them to explain how they can tackle it supported by place value (arrow) cards to demonstrate. Repeat with child choosing further calculations and numbers.
Add and subtract a pair of numbers that involves crossing boundaries, recognising when to adjust, to compensate and to carry numbers across the boundaries. **4a Y6** **4b Y6**	Has difficulty in choosing suitable methods for calculations that cross boundaries: addition ***4a Y6 +/–*** Has difficulty in choosing suitable methods for calculations that cross boundaries: subtraction ***4b Y6 +/–***	What would you add to 37 to make the nearest multiple of 10? What would you add to/subtract from 240 to make the nearest multiple of 100? Explain to me, using an empty number line, how you find 300 – 237. Using an empty number line, explain how you would work out 5016 + 3700. What is 217 – 6? … 217 – 7? What happens when you subtract 8 from 217? Explain how you subtract 18 from 217.	Continue to practise counting up using an empty number line and record the steps, focusing on the significance of the multiples of ten and hundred as milestones to support developing understanding of crossing boundaries. Use place value (arrow) cards to partition before adding and subtracting numbers, to illustrate the significance of columns to support calculation.	Transfer calculation practice to a vertical number line, taking the opportunity to relate this to measures. Increase the number of boundaries to be crossed, asking the child to compare calculation methods considering their efficiency.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Year 4 key objective Carry out column addition and subtraction of two integers less than 1000 and column addition of more than two such integers (NNS *Framework for teaching mathematics*, Supplement of Examples, Section 6, pages 48, 50)

Associated knowledge and skills	**Errors and misconceptions**	**Questions to identify errors and misconceptions**	**Teaching to address the errors and misconceptions**	**Next steps in moving towards the key objective**
Know the value of each digit in a three-digit number, for example that the 3 in 437 has a value of 30. Recognise the significance of each digit in numbers up to 1000 and find an estimate of the addition or subtraction calculation. **1 Y4**	Has insecure understanding of the structure of the number system, resulting in addition and subtraction errors and difficulty with estimating. ***1 Y4 +/–***	Is 43 smaller or larger than 34? What about 343 and 334? How did you decide? Which is the largest/smallest number: 216, 612 or 162? How do you know? What number would you add to 437 to make 477? How did you decide?	Use a range of place value models and images such as bundles of straws, digit cards and structured apparatus to partition numbers and read them correctly. Use a number line and place value (arrow) cards to identify position of a variety of numbers in relation to their nearest ten, nearest hundred. Make two-digit and three-digit numbers using digit cards. Change one digit at a time, for example 134 to 144, establishing the size of the change, in this case ten more. Establish whether the change is significant, for example that 154 is closer to 200 than 100, and use this to inform estimates for addition and subtraction calculations.	From a set of digit cards, ask the child to select three, of which two must be the same, for example 3, 3 and 6. Ask the child to make and read aloud different three-digit numbers, for example three hundred and thirty-six. Repeat activity. Ask the child to estimate answers for some given calculations involving two- and three-digit numbers and discuss with child the estimation methods used.
Partition two- and three-digit numbers in a number of ways. **2 Y4**	Has difficulty in partitioning, for example, 208 into 190 and 18 and 31 into 20 and 11. ***2 Y4 +/–***	What number does 20 + 10 + 1 represent? Is it the same as 20 + 11? What should I add to 190 to make 208? What other ways can you partition 208?	Use place value (arrow) cards to partition and combine numbers with zero as place holder, discussing the position of the zero as a place holder. Use number squares or a string of beads to model the partitioning of numbers in different ways, recording the associated number statements, such as 43 = 40 + 3 = 30 + 13 = 20 + 23	Use partitioning to carry out a range of two-digit calculations, discussing the way partitioning makes the calculation more manageable. Repeat with three-digit numbers.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Associated knowledge skills	Errors and and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Know when it is most appropriate to use column addition or subtraction rather than carrying out the calculation by another method. **3 Y4**	Does not make sensible decisions about when to use calculations laid out in columns. ***3 Y4 +/–***	What is 700 – 1? … 700 – 10? … 700 – 9? What about 30 + 20? … 30 + 21? … 30 + 27? … 296 – 57? … 328 + 187? How did you do these?	Ask the child to choose pairs of numbers to add together, highlighting where mental methods are most appropriate and ones where written methods are more appropriate.	Ask the child to identify a collection of pairs of numbers that would definitely be best tackled by written calculation methods and explain why.
Recognise that the operation of column addition can apply to more than two numbers. **4 Y4**	Has difficulty with adding three numbers in a column, except by adding the first two and then the last one. ***4 Y4 +/–***	Referring to a written addition column calculation involving three numbers, ask the child to talk through their steps in calculation.	Use place value (arrow) cards to demonstrate column addition of more than two numbers and focus on completing the calculation one column at a time.	Extend to adding more than three numbers using a column method.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Year 2 key objective Understand that subtraction is the inverse of addition; state the subtraction corresponding to a given addition and vice versa (NNS *Framework for teaching mathematics*, Supplement of Examples, Section 5, pages 25, 29, 35)

Associated knowledge and skills	**Errors and misconceptions**	**Questions to identify errors and misconceptions**	**Teaching to address the errors and misconceptions**	**Next steps in moving towards the key objective**
Count on and back in ones and tens. **1 Y2**	Makes mistakes when counting using teen numbers and/or crossing boundaries. ***1 Y2 +/–***	Choose a teen number and count backwards from your number in ones. What is the next number, the number before? Count forwards in tens from 7. What number is ten more than 27? … ten less than 97?	Use a number line and a 100-square to count forwards and backwards in ones and tens. Encourage similar counting activities using a visualised number line.	Use known facts about adding and subtracting ten to work out how to calculate using nine and eleven.
Identify pairs of numbers that add to twenty and use known number facts to add mentally. **2 Y2**	Has difficulty in remembering number pairs totalling between ten and twenty, resulting in calculation errors. ***2 Y2 +/–***	What is 3 + 5? What is 13 + 5? How did you work that out? Which pair of numbers adds to 18? Are there any other pairs?	Use rulers and number squares to support practice with recall of number pairs beyond ten and emphasise patterns in number pairs.	Extend patterns to numbers beyond 20; use 4 + 3 = 7 to deduce other number facts such as 14 + 3 = 4 + 13 and so on.
Find a difference by counting up from the smaller to the larger number. **3 Y2**	Counts up unreliably; still counting the smaller number to get one too many in the answer. ***3 Y2 +/–***	How many do I add on to get from three to eight? I've got three sherbet dips and I want eight. How many do I need to buy?	Use a number line to demonstrate counting up by jumping in ones and highlighting the equivalent larger jump. This will illustrate that the starting number position is not included in the jump.	Ask the child to choose a new jump size, for example twos or fives, to find the difference between two given numbers. How can you check your answer?

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Associated knowledge skills	Errors and and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Recognise subtraction as taking away, finding the difference and complementary addition. **4 Y2**	Does not relate finding a difference and complementary addition to the operation of subtraction. ***4 Y2 +/–***	What is the difference between 21 and 18? How did you work it out? What operation did you use? What other ways can you think of for subtracting 18 from 21?	Illustrate the connections between different aspects of subtraction using a range of models and images, such as number lines, money, counters or Interactive Teaching Programmes (ITPs) such as *Counting on and back, Differences, Number facts*.	Provide the following three questions: What is the difference between 21 and 18? 21 – 18 = ? 18 + □ = 21? What do you notice about these and their answers? Make up some of your own and explain the patterns you are using.
Recognise, for example, that subtracting 13 'undoes' adding 13 and vice versa, and that this means that since 4 + 13 = 17, we can state the inverse that 17 – 13 = 4. **5 Y2**	Is insecure in making links between addition and subtraction and/or recognising inverses. ***5 Y2 +/–***	What is the answer to 30 add 20? If 30 add 20 is 50, what is 50 subtract 20? What is 17 subtract 8? Write a number sentence for this calculation. Use the three numbers to write an addition fact.	Use digit cards and number lines to make and check number statements that involve inverse pairs such as 11 + 3 = 14, 14 – 3 = 11.	Give the child statements to sort into pairs involving inverses, then into groups that make up a set of the four equivalent statements such as 24 + 3 = 27, 3 + 24 = 27 27 – 3 = 24, 27 – 24 = 3 Invite the child to make sets of equivalent statements using numbers they have chosen.
Develop and recognise patterns to help deduce other addition and subtraction facts. **6 Y2**	Does not readily use number patterns to support calculating, for example: 46 – 5 = 41, so 46 – 15 = 31, 46 – 25 = 21, etc. ***6 Y2 +/–***	What is 14 + 5? …14 + 15? …14+25? Using this information, tell me what 24 + 25 is. What is 6 – 4? …16 – 4? … 26 – 4? Now what do you think the answer is to 56 – 4? How did you work that out?	Use number squares to demonstrate and highlight patterns such as 5 + 1 = 6, 15 + 1 = 16, 25 + 1 = 26 32 – 2 = 30, 32 – 12 = 20 getting children to predict and check other cases.	Extend to include patterns where the calculations cross a boundary such as 14 + 9 = 23, 14 + 19 = 33 56 – 9 = 47, 56 – 19 = 37.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Reception key objective Begin to relate addition to combining two groups of objects, and subtraction to 'taking away' (NNS *Framework for teaching mathematics*, Supplement of Examples, Section 4, pages 14, 15, 16)

Associated knowledge and skills	**Errors and misconceptions**	**Questions to identify errors and misconceptions**	**Teaching to address the errors and misconceptions**	**Next steps in moving towards the key objective**
Count along and back on a number track to and from a given position. Count objects set out in different arrangements; begin to recognise small numbers without counting and that the number of objects is not affected by their position. Count objects that are out of reach. **1 YR**	Can only begin counting at one; inaccurately counts objects when rearranged; has no consistent recognition of small numbers of objects; lacks systematic approaches. ***1 YR +/–***	Ask the child to choose a starting number. What are the next two numbers? What number comes after the third number? What comes before your starting number? The contents panel says there are twelve. Let's check. Tip them out and put them back. How many are there now? Can you tell me how many sweets there are here without counting them? How many spots are in this picture? Throw a small number of objects onto the table. Can you count them without touching them? How did you do it? How do you know you're correct?	Children walk forwards and backwards along a large number track, along a number line, on a snakes and ladders board, counting aloud. Start from different positions; use digit cards or dice to select start and to move one forwards, two backwards, etc. Using sets of mixed objects to count and rearrange, ask children to estimate and check after each rearrangement. Put small numbers of objects in familiar and unfamiliar patterns and compare with known patterns such as spots on dice, displays on wall, etc. Count sounds such as drum beats or coins dropping into a money box; provide counters or pencils to record marks as they count; count counters or their marks on a sheet; compare ways to systematically count particular arrangements, for example window panes, squares on a grid, chairs around a table.	Cover up selected numbers for children to identify as they count; move forward and backwards from different starting points; count aloud and silently to determine position after a move. Arrange a known number of objects into two or more groups to establish that the total remains the same; count in twos; count objects arranged in pairs; use recognisable patterns of small numbers, such as 3 and 4, to introduce counting in threes and fours. Estimate the number of objects that can be counted reliably; check by counting, first touching objects then without touching.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Associated knowledge skills	Errors and and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Find one more and one less than a given number. **2 YR**	Misunderstands meaning of 'one more' and 'one less'; does not consistently identify the number before or after a given number. ***2 YR +/–***	Here are four counters. How many will you have if I give you one more? There are six spots showing on my dice. Imagine there is one less spot. How many spots would there be? What is one more than seven? … one less than seven?	The child counts aloud from a given starting number, stopping at particular numbers. What number comes next? What number comes before? Relate counting to a set of objects, add one more and ask children to identify how many, similarly one less; arrange objects alongside a number track and keep taking one away to link one less to the number before and similarly one more to the next number.	What is one less than six? …one more than four? Ask me another pair of these questions with the same answer. Count every other number; identify odd and even numbers; count in twos; add and take away two objects and find two more and two less than a given number; build on pattern recognition to introduce three more, three less, etc.
Say how many there are altogether by counting all the objects when combining groups for addition. Separate a given number of objects into two or more groups and say how many there are in each group. **3 YR**	Does not relate the combining of groups of objects to addition and/or does not interpret the counting of all of the objects as an answer to the question 'How many are there altogether?' ***3 YR +/–***	How many spots are there on this blue card? How many spots are there on this red card? How many spots are there altogether? There are three spoons in this cup and two spoons in this cup. How many spoons altogether? How do you know? Listen to these claps. How many were there? Now tell me how many extra claps I make. How many claps is that altogether? There are seven grapes in my lunch box. How many red grapes and green grapes could there be? Use these cubes to show how you did it.	Use containers, such as an egg box, that hold a given number of objects. Give the child groups of objects to count, for example four objects and two objects, and place the objects in the container for children to count all the objects. Hide the container and ask how many objects it holds. Recount, check and confirm. Repeat with other number pairs. Arrange a small number of biscuits on two plates. Ask the child to count the biscuits on each plate and say how many there are altogether. Move biscuits between the plates and repeat activity.	Arrange a small number of biscuits on two plates so that one plate has two more biscuits than the other. Ask the child to say how many biscuits on each plate and how many altogether. Repeat with different numbers of biscuits as appropriate. Extend to three plates.

Tracking children's learning through the *NNS Framework for teaching mathematics* (addition and subtraction)

Associated knowledge skills	Errors and and misconceptions	Questions to identify errors and misconceptions	Teaching to address the errors and misconceptions	Next steps in moving towards the key objective
Say how many are left when some objects are taken away, by counting how many objects are left. **4 YR**	Is not confident about when to stop counting when taking away (subtracting) in answer to the question 'How many are left?' ***4 YR +/–***	There are six in this box. I take away two and put them here. How many are left? Here are eight cups. Three cups are full and the rest are empty. How many are empty? We ate four of the six cakes we made. How many are left? How did you work this out?	Use a range of objects and resources such as beads on a string. Invite the child to take one away and count the rest. How many are left? Repeat the activity, inviting the child to take away different numbers and describe what has happened. Emphasise that it is the number of objects left, not the objects that we take away, that is important.	Continue to ask questions that involve finding how many are left after taking away or subtracting; ask the child to find their own way of recording their answers and to describe their solutions using a range of vocabulary such as: 6 take away 4 is 2 6 subtract 4 is 2 Start with a number and take a number away. How many are left? Ask the child to start with another number and take some away to leave the same number as before. Record the calculations before repeating the activity. Ask the child to comment on what they notice and explain how they chose their numbers.